LEARNING Haskell Language

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#haskell
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About

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Chapter 1: Getting started with Haskell

Language

Remarks

Haskell

Haskell is an advanced purely-functional programming language.

Features:

- **Statically typed**: Every expression in Haskell has a type which is determined at compile time. Static type checking is the process of verifying the type safety of a program based on analysis of a program’s text (source code). If a program passes a static type checker, then the program is guaranteed to satisfy some set of type safety properties for all possible inputs.
- **Purely functional**: Every function in Haskell is a function in the mathematical sense. There are no statements or instructions, only expressions which cannot mutate variables (local or global) nor access state like time or random numbers.
- **Concurrent**: Its flagship compiler, GHC, comes with a high-performance parallel garbage collector and light-weight concurrency library containing a number of useful concurrency primitives and abstractions.
- **Lazy evaluation**: Functions don’t evaluate their arguments. Delays the evaluation of an expression until its value is needed.
- **General-purpose**: Haskell is built to be used in all contexts and environments.
- **Packages**: Open source contribution to Haskell is very active with a wide range of packages available on the public package servers.

The latest standard of Haskell is Haskell 2010. As of May 2016, a group is working on the next version, Haskell 2020.

The official Haskell documentation is also a comprehensive and useful resource. Great place to find books, courses, tutorials, manuals, guides, etc.

Versions

<table>
<thead>
<tr>
<th>Version</th>
<th>Release Date</th>
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<tr>
<td>Haskell 2010</td>
<td>2012-07-10</td>
</tr>
<tr>
<td>Haskell 98</td>
<td>2002-12-01</td>
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https://riptutorial.com/
Examples

Hello, World!

A basic "Hello, World!" program in Haskell can be expressed concisely in just one or two lines:

```haskell
main :: IO ()
main = putStrLn "Hello, World!"
```

The first line is an optional type annotation, indicating that `main` is a value of type `IO ()`, representing an I/O action which "computes" a value of type `()` (read "unit"); the empty tuple conveying no information) besides performing some side effects on the outside world (here, printing a string at the terminal). This type annotation is usually omitted for `main` because it is its only possible type.

Put this into a `helloworld.hs` file and compile it using a Haskell compiler, such as GHC:

```
ghc helloworld.hs
```

Executing the compiled file will result in the output "Hello, World!" being printed to the screen:

```
./helloworld
Hello, World!
```

Alternatively, `runhaskell` or `runghc` make it possible to run the program in interpreted mode without having to compile it:

```
runhaskell helloworld.hs
```

The interactive REPL can also be used instead of compiling. It comes shipped with most Haskell environments, such as `ghci` which comes with the GHC compiler:

```
ghci> putStrLn "Hello World!"
Hello, World!
ghci>
```

Alternatively, load scripts into `ghci` from a file using `load` (or `:l`):

```
ghci> :load helloworld
```

`:reload` (or `:r`) reloads everything in `ghci`:

```
Prelude> :l helloworld.hs
[1 of 1] Compiling Main            ( helloworld.hs, interpreted )

<some time later after some edits>
*Main> :r
```
Ok, modules loaded: Main.

**Explanation:**

This first line is a type signature, declaring the type of `main`:

```
main :: IO ()
```

Values of type `IO ()` describe actions which can interact with the outside world.

Because Haskell has a fully-fledged Hindley-Milner type system which allows for automatic type inference, type signatures are technically optional: if you simply omit the `main :: IO ()`, the compiler will be able to infer the type on its own by analyzing the definition of `main`. However, it is very much considered bad style not to write type signatures for top-level definitions. The reasons include:

- Type signatures in Haskell are a very helpful piece of documentation because the type system is so expressive that you often can see what sort of thing a function is good for simply by looking at its type. This “documentation” can be conveniently accessed with tools like GHCi. And unlike normal documentation, the compiler's type checker will make sure it actually matches the function definition!

- Type signatures *keep bugs local*. If you make a mistake in a definition without providing its type signature, the compiler may not immediately report an error but instead simply infer a nonsensical type for it, with which it actually typechecks. You may then get a cryptic error message when *using* that value. With a signature, the compiler is very good at spotting bugs right where they happen.

This second line does the actual work:

```
main = putStrLn "Hello, World!"
```

If you come from an imperative language, it may be helpful to note that this definition can also be written as:

```
main = do
    putStrLn "Hello, World!" ;
    return ()
```

Or equivalently (Haskell has layout-based parsing; but *beware mixing tabs and spaces inconsistently* which will confuse this mechanism):

```
main = do
    putStrLn "Hello, World!"
    return ()
```
Each line in a `do` block represents some monadic (here, I/O) *computation*, so that the whole `do` block represents the overall action comprised of these sub-steps by combining them in a manner specific to the given monad (for I/O this means just executing them one after another).

The `do` syntax is itself a syntactic sugar for monads, like `IO` here, and `return` is a no-op action producing its argument without performing any side effects or additional computations which might be part of a particular monad definition.

The above is the same as defining `main = putStrLn "Hello, World!"`, because the value `putStrLn "Hello, World!"` already has the type `IO ()`. Viewed as a “statement”, `putStrLn "Hello, World!"` can be seen as a complete program, and you simply define `main` to refer to this program.

You can [look up the signature of `putStrLn` online](https://hackage.haskell.org/package/base-4.12.0.0/docs/IO.html#v:putStrLn):

```haskell
putStrLn :: String -> IO ()
-- thus,
putStrLn (v :: String) :: IO ()
```

`putStrLn` is a function that takes a string as its argument and outputs an I/O-action (i.e. a value representing a program that the runtime can execute). The runtime always executes the action named `main`, so we simply need to define it as equal to `putStrLn "Hello, World!"`.

**Factorial**

The factorial function is a Haskell "Hello World!" (and for functional programming generally) in the sense that it succinctly demonstrates basic principles of the language.

**Variation 1**

```haskell
fac :: (Integral a) => a -> a
fac n = product [1..n]
```

`Integral` is the class of integral number types. Examples include `Int` and `Integer`.

- `(Integral a) =>` places a constraint on the type `a` to be in said class
- `fac :: a -> a` says that `fac` is a function that takes an `a` and returns an `a`
- `product` is a function that accumulates all numbers in a list by multiplying them together.
- `[1..n]` is special notation which desugars to `enumFromTo 1 n`, and is the range of numbers $1 \leq x \leq n$.

**Variation 2**

```haskell
fac :: (Integral a) => a -> a
fac 0 = 1
fac n = n * fac (n - 1)
```
Live demo

This variation uses pattern matching to split the function definition into separate cases. The first definition is invoked if the argument is 0 (sometimes called the stop condition) and the second definition otherwise (the order of definitions is significant). It also exemplifies recursion as $\text{fac}$ refers to itself.

It is worth noting that, due to rewrite rules, both versions of $\text{fac}$ will compile to identical machine code when using GHC with optimizations activated. So, in terms of efficiency, the two would be equivalent.

Fibonacci, Using Lazy Evaluation

Lazy evaluation means Haskell will evaluate only list items whose values are needed.

The basic recursive definition is:

\[
\begin{align*}
\text{f (0)} & \leftarrow 0 \\
\text{f (1)} & \leftarrow 1 \\
\text{f (n)} & \leftarrow \text{f (n-1)} + \text{f (n-2)}
\end{align*}
\]

If evaluated directly, it will be very slow. But, imagine we have a list that records all the results,

\[
\text{fibs !! n} \leftarrow \text{f (n)}
\]

Then

\[
\begin{array}{c}
\text{fibs} \rightarrow 0 : 1 : f(0) + f(1) : f(1) + f(2) : f(2) + f(3) : \ldots \\
\end{array}
\]

\[
\begin{array}{c}
\rightarrow 0 : 1 : f(1) + f(2) : f(2) + f(3) : \ldots \\
\end{array}
\]

This is coded as:

\[
\begin{align*}
\text{fibn} \ n & = \text{fibs !! n} \\
\text{where} \\
\text{fibs} & = 0 : 1 : \text{map} \ f \ [2..] \\
\text{f n} & = \text{fibs !! (n-1)} + \text{fibs !! (n-2)}
\end{align*}
\]

Or even as

https://riptutorial.com/
GHCi> let fibs = 0 : zipWith (+) fibs (tail fibs)
GHCi> take 10 fibs
[0, 1, 1, 2, 3, 5, 8, 13, 21, 34]

zipWith makes a list by applying a given binary function to corresponding elements of the two lists given to it, so zipWith (+) [x₁, x₂, ...] [y₁, y₂, ...] is equal to [x₁ + y₁, x₂ + y₂, ...].

Another way of writing fibs is with the scanl function:

GHCi> let fibs = 0 : scanl (+) 1 fibs
GHCi> take 10 fibs
[0, 1, 1, 2, 3, 5, 8, 13, 21, 34]

scanl builds the list of partial results that foldl would produce, working from left to right along the input list. That is, scanl f z₀ [x₁, x₂, ...] is equal to [z₀, z₁, z₂, ...] where z₁ = f z₀ x₁; z₂ = f z₁ x₂; ...

Thanks to lazy evaluation, both functions define infinite lists without computing them out entirely. That is, we can write a fib function, retrieving the nth element of the unbounded Fibonacci sequence:

GHCi> let fib n = fibs !! n  -- (!!) being the list subscript operator
  -- or in point-free style:
GHCi> let fib = (fibs !!)
GHCi> fib 9
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Getting started

Online REPL

The easiest way to get started writing Haskell is probably by going to the Haskell website or Try Haskell and use the online REPL (read-eval-print-loop) on the home page. The online REPL supports most basic functionality and even some IO. There is also a basic tutorial available which can be started by typing the command help. An ideal tool to start learning the basics of Haskell and try out some stuff.

GHC(i)

For programmers that are ready to engage a little bit more, there is GHCi, an interactive environment that comes with the Glorious/Glasgow Haskell Compiler. The GHC can be installed separately, but that is only a compiler. In order to be able to install new libraries, tools like Cabal and Stack must be installed as well. If you are running a Unix-like operating system, the easiest installation is to install Stack using:

curl -sSL https://get.haskellstack.org/ | sh
This installs GHC isolated from the rest of your system, so it is easy to remove. All commands must be preceded by `stack` though. Another simple approach is to install a Haskell Platform. The platform exists in two flavours:

1. The **minimal** distribution contains only GHC (to compile) and Cabal/Stack (to install and build packages)
2. The **full** distribution additionally contains tools for project development, profiling and coverage analysis. Also an additional set of widely-used packages is included.

These platforms can be installed by downloading an installer and following the instructions or by using your distribution’s package manager (note that this version is not guaranteed to be up-to-date):

- **Ubuntu, Debian, Mint:**
  
  sudo apt-get install haskell-platform

- **Fedora:**
  
  sudo dnf install haskell-platform

- **Redhat:**
  
  sudo yum install haskell-platform

- **Arch Linux:**
  
  sudo pacman --prefix $ghc cabal-install haskell-haddock-api \  haskell-haddock-library happy alex

- **Gentoo:**
  
  sudo layman --prefix / \  -a haskell
  sudo emerge haskell-platform

- **OSX with Homebrew:**
  
  brew cask install haskell-platform

- **OSX with MacPorts:**
  
  sudo port install haskell-platform

Once installed, it should be possible to start GHCi by invoking the `ghci` command anywhere in the terminal. If the installation went well, the console should look something like

```
me@notebook:~$ ghci
```
possibly with some more information on what libraries have been loaded before the Prelude>. Now, the console has become a Haskell REPL and you can execute Haskell code as with the online REPL. In order to quit this interactive environment, one can type :q or :quit. For more information on what commands are available in GHCi, type :? as indicated in the starting screen.

Because writing the same things again and again on a single line is not always that practically, it might be a good idea to write the Haskell code in files. These files normally have .hs for an extension and can be loaded into the REPL by using :l or :load.

As mentioned earlier, GHCi is a part of the GHC, which is actually a compiler. This compiler can be used to transform a .hs file with Haskell code into a running program. Because a .hs file can contain a lot of functions, a main function must be defined in the file. This will be the starting point for the program. The file test.hs can be compiled with the command

```
ghc test.hs
```

this will create object files and an executable if there were no errors and the main function was defined correctly.

### More advanced tools

1. It has already been mentioned earlier as package manager, but stack can be a useful tool for Haskell development in completely different ways. Once installed, it is capable of

   - installing (multiple versions of) GHC
   - project creation and scaffolding
   - dependency management
   - building and testing projects
   - benchmarking

2. IHaskell is a haskell kernel for IPython and allows to combine (runnable) code with markdown and mathematical notation.

### Primes

A few most salient variants:

### Below 100

```haskell
import Data.List (\)

ps100 = ((\[2, 100\] \ [4, 100]) \ [6, 100]) \ [10, 100]) \ [14, 100])
-- = ((\[2, 100\] \ [4, 100]) \ [6, 100]) \ [10, 100]) \ [14, 100])
```

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Unlimited

Sieve of Eratosthenes, using data-ordlist package:

```haskell
import qualified Data.List.Ordered

ps   = 2 : _Y ((3:) . minus [5,7..] . unionAll . map (\p -> [p*p, p*p+2*p..]))

_Y g = g (_Y g)  -- = g (g (g (g (g (...)))) = g . g . g . g...

```

Traditional

(a sub-optimal trial division sieve)

```haskell
ps = sieve [2..]
where
    sieve (x:xs) = [x] ++ sieve [y | y <- xs, rem y x > 0]

-- = map head ( iterate (\(x:xs) -> filter ((> 0).(`rem` x)) xs) [2..])
```

Optimal trial division

```haskell
ps = 2 : [n | n <- [3..], all ((> 0).rem n) $ takeWhile ((<= n).(^2)) ps]

-- = 2 : [n | n <- [3..], foldr (\p r-> p*p > n || (rem n p > 0 && r)) True ps]
```

Transitional

From trial division to sieve of Eratosthenes:

```
[n | n <- [2..], []==[i | i <= [2..n-1], j <= [0,i..n], j==n]]
```

The Shortest Code

```haskell
nubBy ((>1).).gcd) [2..]  -- i.e., nubBy (\a b -> gcd a b > 1) [2..]
```

nubBy is also from Data.List, like (\\).

Declaring Values

We can declare a series of expressions in the REPL like this:

```
Prelude> let x = 5
Prelude> let y = 2 * 5 + x
Prelude> let result = y * 10
Prelude> x
```

https://riptutorial.com/
To declare the same values in a file we write the following:

```haskell
-- demo.hs
module Demo where
-- We declare the name of our module so
-- it can be imported by name in a project.
x = 5
y = 2 * 5 + x
result = y * 10
```

Module names are capitalized, unlike variable names.

Read Getting started with Haskell Language online:
Chapter 2: Applicative Functor

Introduction

Applicative is the class of types f :: * -> * which allows lifted function application over a structure where the function is also embedded in that structure.

Remarks

Definition

```haskell
class Functor f => Applicative f where
  pure  :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

Note the `Functor` constraint on `f`. The `pure` function returns its argument embedded in the Applicative structure. The infix function `<*>` (pronounced "apply") is very similar to `fmap` except with the function embedded in the Applicative structure.

A correct instance of `Applicative` should satisfy the **applicative laws**, though these are not enforced by the compiler:

```haskell
pure id <*> a = a                              -- identity
pure (.) <*> a <*> b <*> c = a <*> (b <*> c)   -- composition
pure f <*> pure a = pure (f a)                 -- homomorphism
a <*> pure b = pure ($ b) <*> a                -- interchange
```

Examples

Alternative definition

Since every Applicative Functor is a `Functor`, `fmap` can always be used on it; thus the essence of Applicative is the pairing of carried contents, as well as the ability to create it:

```haskell
class Functor f => PairingFunctor f where
  funit :: f ()                  -- create a context, carrying nothing of import
  fpair :: (f a,f b) -> f (a,b)  -- collapse a pair of contexts into a pair-carrying context
```

This class is isomorphic to `Applicative`.

```haskell
pure a = const a <$> funit = a <$> funit
fa <*> fb = (\(a,b) -> a b) <$> fpair (fa, fb) = uncurry ($) <$> fpair (fa, fb)
```

Conversely,
funit = pure ()

fpair (fa, fb) = (,) <$> fa <*> fb

Common instances of Applicative

---

**Maybe**

**Maybe** is an applicative functor containing a possibly-absent value.

```haskell
instance Applicative Maybe where
    pure = Just
    Just f <*> Just x = Just $ f x
    _ <*> _ = Nothing
```

`pure` lifts the given value into `Maybe` by applying `Just` to it. The `(<*>)` function applies a function wrapped in a `Maybe` to a value in a `Maybe`. If both the function and the value are present (constructed with `Just`), the function is applied to the value and the wrapped result is returned. If either is missing, the computation can't proceed and `Nothing` is returned instead.

---

**Lists**

One way for lists to fit the type signature `(<*>) :: [a -> b] -> [a] -> [b]` is to take the two lists' Cartesian product, pairing up each element of the first list with each element of the second one:

```haskell
fs <*> xs = [f x | f <- fs, x <- xs]
```

This is usually interpreted as emulating nondeterminism, with a list of values standing for a nondeterministic value whose possible values range over that list; so a combination of two nondeterministic values ranges over all possible combinations of the values in the two lists:

```haskell
ghci> [(+1),(+2)] <*> [3,30,300]
[4,31,301,5,32,302]
```

---

**Infinite streams and zip-lists**

There's a class of Applicatives which "zip" their two inputs together. One simple example is that of infinite streams:

```haskell
data Stream a = Stream { headS :: a, tailS :: Stream a }
```
Stream's Applicative instance applies a stream of functions to a stream of arguments point-wise, pairing up the values in the two streams by position. pure returns a constant stream — an infinite list of a single fixed value:

```haskell
instance Applicative Stream where
  pure x = let s = Stream x s in s
  Stream f fs <*> Stream x xs = Stream (f x) (fs <*> xs)
```

Lists too admit a "zippy" Applicative instance, for which there exists the ZipList newtype:

```haskell
newtype ZipList a = ZipList { getZipList :: [a] }
instance Applicative ZipList where
  ZipList xs <*> ZipList ys = ZipList $ zipWith ($) xs ys
```

Since zip trims its result according to the shortest input, the only implementation of pure that satisfies the Applicative laws is one which returns an infinite list:

```haskell
pure a = ZipList (repeat a)  -- ZipList (fix (a:)) = ZipList [a,a,a,a,...
```

For example:

```haskell
ghci> getZipList $ ZipList [(+1),(+2)] <*> ZipList [3,30,300]
[4,32]
```

The two possibilities remind us of the outer and the inner product, similar to multiplying a 1-column (n x 1) matrix with a 1-row (1 x m) one in the first case, getting the n x m matrix as a result (but flattened); or multiplying a 1-row and a 1-column matrices (but without the summing up) in the second case.

---

### Functions

When specialised to functions (->) r, the type signatures of pure and <*> match those of the K and S combinators, respectively:

```haskell
pure :: a -> (r -> a)
<*> :: (r -> (a -> b)) -> (r -> a) -> (r -> b)
```

pure must be const, and <*> takes a pair of functions and applies them each to a fixed argument, applying the two results:

```haskell
instance Applicative ((->) r) where
  pure = const
  f <*> g = \x -> f x (g x)
```

Functions are the prototypical "zippy" applicative. For example, since infinite streams are isomorphic to (->) Nat, ...

https://riptutorial.com/
-- | Index into a stream
\[
\text{to} \colon \text{Stream} \ a \rightarrow (\text{Nat} \rightarrow a)
\]
\[
\text{to} \ (\text{Stream} \ x \ \text{xs}) \ \text{Zero} = x
\]
\[
\text{to} \ (\text{Stream} \ x \ \text{xs}) \ (\text{Suc} \ n) = \text{to} \ \text{xs} \ n
\]

-- | List all the return values of the function in order
\[
\text{from} \colon (\text{Nat} \rightarrow a) \rightarrow \text{Stream} \ a
\]
\[
\text{from} \ f = \text{from'} \ \text{Zero}
\]
\[
\quad \quad \text{where} \ \text{from'} \ n = \text{Stream} \ (f \ n) \ (\text{from'} \ (\text{Suc} \ n))
\]

... representing streams in a higher-order way produces the zippy Applicative instance automatically.

Read Applicative Functor online: https://riptutorial.com/haskell/topic/8162/applicative-functor
Chapter 3: Arbitrary-rank polymorphism with RankNTypes

Introduction

GHC’s type system supports arbitrary-rank explicit universal quantification in types through the use of the Rank2Types and RankNTypes language extensions.

Syntax

- Arbitrary rank quantification is enabled with either the Rank2Types or RankNTypes language extension.
- With this extension enabled, the `forall` keyword can be used to add higher-rank quantification.

Examples

RankNTypes

StackOverflow forces me to have one example. If this topic is approved, we should move this example here.

Read Arbitrary-rank polymorphism with RankNTypes online: https://riptutorial.com/haskell/topic/8984/arbitrary-rank-polymorphism-with-rankntypes
Chapter 4: Arithmetic

Introduction

In Haskell, all expressions (which includes numerical constants and functions operating on those) have a decidable type. At compile time, the type-checker infers the type of an expression from the types of the elementary functions that compose it. Since data is immutable by default, there are no "type casting" operations, but there are functions that copy data and generalize or specialize the types within reason.

Remarks

The numeric typeclass hierarchy

`Num` sits at the root of the numeric typeclass hierarchy. Its characteristic operations and some common instances are shown below (the ones loaded by default with Prelude plus those of `Data.Complex`):

```
λ> :i Num
class Num a where
  (+) :: a -> a -> a
  (\-) :: a -> a -> a
  (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
{-# MINIMAL (+), (*), abs, signum, fromInteger, (negate | (-)) #-}
instance RealFloat a => Num (Complex a) -- Defined in 'Data.Complex'
instance Num Word -- Defined in 'GHC.Num'
instance Num Integer -- Defined in 'GHC.Num'
instance Num Int -- Defined in 'GHC.Num'
instance Num Float -- Defined in 'GHC.Float'
instance Num Double -- Defined in 'GHC.Float'
```

We have already seen the `Fractional` class, which requires `Num` and introduces the notions of "division" `/` and reciprocal of a number:

```
λ> :i Fractional
class Fractional a where
  (/) :: a -> a -> a
  recip :: a -> a
  fromRational :: Rational -> a
{-# MINIMAL fromRational, (recip | (/)) #-}
instance RealFloat a => Fractional (Complex a) -- Defined in 'Data.Complex'
instance Fractional Float -- Defined in 'GHC.Float'
instance Fractional Double -- Defined in 'GHC.Float'
```

https://riptutorial.com/
The `Real` class models the real numbers. It requires `Num` and `Ord`, therefore it models an ordered numerical field. As a counterexample, Complex numbers are *not* an ordered field (i.e. they do not possess a natural ordering relationship):

```
λ> :i Real
class (Num a, Ord a) => Real a where
  toRational :: a -> Rational
{-# MINIMAL toRational #-}
  -- Defined in 'GHC.Real'
instance Real Word -- Defined in 'GHC.Real'
instance Real Integer -- Defined in 'GHC.Real'
instance Real Int -- Defined in 'GHC.Real'
instance Real Float -- Defined in 'GHC.Float'
instance Real Double -- Defined in 'GHC.Float'
```

`RealFrac` represents numbers that may be rounded.

```
λ> :i RealFrac
class (Real a, Fractional a) => RealFrac a where
  properFraction :: Integral b => a -> (b, a)
  truncate :: Integral b => a -> b
  round :: Integral b => a -> b
  ceiling :: Integral b => a -> b
  floor :: Integral b => a -> b
{-# MINIMAL properFraction #-}
  -- Defined in 'GHC.Real'
instance RealFrac Float -- Defined in 'GHC.Float'
instance RealFrac Double -- Defined in 'GHC.Float'
```

`Floating` (which implies `Fractional`) represents constants and operations that may not have a finite decimal expansion.

```
λ> :i Floating
class Fractional a => Floating a where
  pi :: a
  exp :: a -> a
  log :: a -> a
  sqrt :: a -> a
  (***) :: a -> a -> a
  logBase :: a -> a -> a
  sin :: a -> a
  cos :: a -> a
  tan :: a -> a
  asin :: a -> a
  acos :: a -> a
  atan :: a -> a
  sinh :: a -> a
  cosh :: a -> a
  tanh :: a -> a
  asinh :: a -> a
  acosh :: a -> a
  atanh :: a -> a
  GHC.Float.log1p :: a -> a
  GHC.Float.expm1 :: a -> a
  GHC.Float.log1pexp :: a -> a
  GHC.Float.log1mexp :: a -> a
{-# MINIMAL pi, exp, log, sin, cos, asin, acos, atan, sinh, cosh,
```

https://riptutorial.com/
Caution: while expressions such as `sqrt . negate :: Floating a => a -> a` are perfectly valid, they might return `NaN` ("not-a-number"), which may not be an intended behaviour. In such cases, we might want to work over the Complex field (shown later).

Examples

Basic examples

```haskell
λ> :t 1
1 :: Num t => t

λ> :t π
π :: Floating a => a
```

In the examples above, the type-checker infers a type-class rather than a concrete type for the two constants. In Haskell, the `Num` class is the most general numerical one (since it encompasses integers and reals), but π must belong to a more specialized class, since it has a nonzero fractional part.

```haskell
list₀ :: [Integer]
list₀ = [1, 2, 3]

list₁ :: [Double]
list₁ = [1, 2, π]
```

The concrete types above were inferred by GHC. More general types like `list₀ :: Num a => [a]` would have worked, but would have also been harder to preserve (e.g. if one consed a `Double` onto a list of `Num`s), due to the caveats shown above.

`Could not deduce (Fractional Int) ...`

The error message in the title is a common beginner mistake. Let’s see how it arises and how to fix it.

Suppose we need to compute the average value of a list of numbers; the following declaration would seem to do it, but it wouldn’t compile:

```haskell
averageOfList ll = sum ll / length ll
```

The problem is with the division (/) function: its signature is `(/) :: Fractional a => a -> a -> a`, but in the case above the denominator (given by `length :: Foldable t => t a -> Int`) is of type `Int` (and `Int` does not belong to the `Fractional` class) hence the error message.
We can fix the error message with `fromIntegral :: (Num b, Integral a) => a -> b`. One can see that this function accepts values of any `Integral` type and returns corresponding ones in the `Num` class:

```
averageOfList' :: (Foldable t, Fractional a) => t a -> a
averageOfList' ll = sum ll / fromIntegral (length ll)
```

**Function examples**

What's the type of `(+)`?

```
λ> :t (+)
(+) :: Num a => a -> a -> a
```

What's the type of `sqrt`?

```
λ> :t sqrt
sqrt :: Floating a => a -> a
```

What's the type of `sqrt . fromIntegral`?

```
sqrt . fromIntegral :: (Integral a, Floating c) => a -> c
```

Read Arithmetic online: [https://riptutorial.com/haskell/topic/8616/arithmetic](https://riptutorial.com/haskell/topic/8616/arithmetic)
Chapter 5: Arrows

Examples

Function compositions with multiple channels

`Arrow` is, vaguely speaking, the class of morphisms that compose like functions, with both serial composition and “parallel composition”. While it is most interesting as a generalisation of functions, the `Arrow (->)` instance itself is already quite useful. For instance, the following function:

```haskell
spaceAround :: Double -> [Double] -> Double
spaceAround x ys = minimum greater - maximum smaller
  where (greater, smaller) = partition (>x) ys
```

can also be written with arrow combinators:

```haskell
spaceAround x = partition (>x) >>> minimum *** maximum >>> uncurry (-)
```

This kind of composition can best be visualised with a diagram:

```
    ─── minimum ───
     /           *            \
    ─── partition (>x) >>>        *        >>>  uncurry (-) ───
     \           *            /
    ─── maximum ───
```

Here,

- The `>>>` operator is just a flipped version of the ordinary . composition operator (there’s also a `<<<` version that composes right-to-left). It pipes the data from one processing step to the next.

- the out-going / \ indicate the data flow is split up in two “channels”. In terms of Haskell types, this is realised with tuples:

  ```haskell
  partition (>x) :: [Double] -> ([Double], [Double])
  ```

  splits up the flow in two `[Double]` channels, whereas

  ```haskell
  uncurry (-) :: (Double, Double) -> Double
  ```

  merges two `Double` channels.

- `***` is the parallel composition operator. It lets `maximum` and `minimum` operate independently on different channels of the data. For functions, the signature of this operator is
At least in the Hask category (i.e. in the Arrow \(\rightarrow\) instance), \(f***g\) does not actually compute \(f\) and \(g\) in parallel as in, on different threads. This would theoretically be possible, though.

Read Arrows online: https://riptutorial.com/haskell/topic/4912/arrows
**Chapter 6: Attoparsec**

**Introduction**

Attoparsec is a parsing combinator library that is "aimed particularly at dealing efficiently with network protocols and complicated text/binary file formats".

Attoparsec offers not only speed and efficiency, but backtracking and incremental input.

Its API closely mirrors that of another parser combinator library, Parsec.

There are submodules for compatibility with 

- ByteString
- Text
- Char8

Use of the OverloadedStrings language extension is recommended.

**Parameters**

<table>
<thead>
<tr>
<th>Type</th>
<th>Detail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parser i  a</td>
<td>The core type for representing a parser. i is the string type, e.g. ByteString.</td>
</tr>
<tr>
<td>IResult i  r</td>
<td>The result of a parse, with Fail i [String] String, Partial (i -&gt; IResult i r) and Done i r as constructors.</td>
</tr>
</tbody>
</table>

**Examples**

**Combinators**

Parsing input is best achieved through larger parser functions that are composed of smaller, single purpose ones.

Let's say we wished to parse the following text which represents working hours:

Monday: 0800 1600.

We could split these into two "tokens": the day name -- "Monday" -- and a time portion "0800" to "1600".

To parse a day name, we could write the following:

```haskell
data Day = Day String

day :: Parser Day
day = do
  name <- takeWhile1 (/= ':')
  skipMany1 (char ':')
  skipSpace
```

https://riptutorial.com/
To parse the time portion we could write:

```haskell
data TimePortion = TimePortion String String

time = do
    start <- takeWhile1 isDigit
    skipSpace
    end <- takeWhile1 isDigit
    return $ TimePortion start end
```

Now we have two parsers for our individual parts of the text, we can combine these in a "larger" parser to read an entire day's working hours:

```haskell
data WorkPeriod = WorkPeriod Day TimePortion

work = do
    d <- day
    t <- time
    return $ WorkPeriod d t
```

and then run the parser:

```haskell
parseOnly work "Monday: 0800 1600"
```

**Bitmap - Parsing Binary Data**

Attoparsec makes parsing binary data trivial. Assuming these definitions:

```haskell
import           Data.Attoparsec.ByteString (Parser, eitherResult, parse, take)
import           Data.Binary.Get            (getWord32le, runGet)
import           Data.ByteString            (ByteString, readFile)
import           Data.ByteString.Char8      (unpack)
import           Data.ByteString.Lazy       (fromStrict)
import           Prelude                    hiding (readFile, take)

-- The DIB section from a bitmap header
data DIB = BM | BA | CI | CP | IC | PT
    deriving (Show, Read)

-- The entire bitmap header
data Header = Header DIB Int Reserved Reserved Int
    deriving (Show)
```

We can parse the header from a bitmap file easily. Here, we have 4 parser functions that represent the header section from a bitmap file:

```haskell
import           Data.Attoparsec.ByteString (Parser, eitherResult, parse, take)
import           Data.Binary.Get            (getWord32le, runGet)
import           Data.ByteString            (ByteString, readFile)
import           Data.ByteString.Char8      (unpack)
import           Data.ByteString.Lazy       (fromStrict)
import           Prelude                    hiding (readFile, take)

-- The DIB section from a bitmap header
data DIB = BM | BA | CI | CP | IC | PT
    deriving (Show, Read)

-- The entire bitmap header
data Header = Header DIB Int Reserved Reserved Int
    deriving (Show)
```

Firstly, the DIB section can be read by taking the first 2 bytes
Similarly, the size of the bitmap, the reserved sections and the pixel offset can be read easily too:

```haskell
sizeP :: Parser Int
sizeP = fromIntegral . runGet getWord32le . fromStrict <$> take 4

reservedP :: Parser Reserved
reservedP = take 2

addressP :: Parser Int
addressP = fromIntegral . runGet getWord32le . fromStrict <$> take 4
```

which can then be combined into a larger parser function for the entire header:

```haskell
bitmapHeader :: Parser Header
bitmapHeader = do
  dib <- dibP
  sz <- sizeP
  reservedP
  reservedP
  offset <- addressP
  return $ Header dib sz "" "" offset
```

Read Attoparsec online: https://riptutorial.com/haskell/topic/9681/attoparsec
Chapter 7: Bifunctor

Syntax

- $\text{bimap} :: (a \rightarrow b) \rightarrow (c \rightarrow d) \rightarrow p\ a\ c \rightarrow p\ b\ d$
- $\text{first} :: (a \rightarrow b) \rightarrow p\ a\ c \rightarrow p\ b\ c$
- $\text{second} :: (b \rightarrow c) \rightarrow p\ a\ b \rightarrow p\ a\ c$

Remarks

A run of the mill Functor is covariant in a single type parameter. For instance, if $f$ is a Functor, then given an $f\ a$, and a function of the form $a \rightarrow b$, one can obtain an $f\ b$ (through the use of $\text{fmap}$).

A Bifunctor is covariant in two type parameters. If $f$ is a Bifunctor, then given an $f\ a\ b$, and two functions, one from $a \rightarrow c$, and another from $b \rightarrow d$, then one can obtain an $f\ c\ d$ (using $\text{bimap}$).

$\text{first}$ should be thought of as an $\text{fmap}$ over the first type parameter, $\text{second}$ as an $\text{fmap}$ over the second, and $\text{bimap}$ should be conceived as mapping two functions covariantly over the first and second type parameters, respectively.

Examples

Common instances of Bifunctor

Two-element tuples

(,) is an example of a type that has a Bifunctor instance.

instance Bifunctor (,) where
  bimap f g (x, y) = (f x, g y)

$bimap$ takes a pair of functions and applies them to the tuple’s respective components.

bimap (+ 2) (++) "nie") (3, "john") --> (5,"johnnie")
bimap ceiling length (3.5 :: Double, "john" :: String) --> (4,4)

Either

Either’s instance of Bifunctor selects one of the two functions to apply depending on whether the value is Left or Right.

instance Bifunctor Either where
  bimap f g (Left x) = Left (f x)
  bimap f g (Right y) = Right (g y)
first and second

If mapping covariantly over only the first argument, or only the second argument, is desired, then `first` or `second` ought to be used (in lieu of `bimap`).

```
first :: Bifunctor f => (a -> c) -> f a b -> f c b
first f = bimap f id

second :: Bifunctor f => (b -> d) -> f a b -> f a d
second g = bimap id g
```

For example,

```
ghci> second (+ 2) (Right 40)
Right 42
ghci> second (+ 2) (Left "uh oh")
Left "uh oh"
```

Definition of Bifunctor

Bifunctor is the class of types with two type parameters `f :: * -> * -> *`, both of which can be covariantly mapped over simultaneously.

```
class Bifunctor f where
    bimap :: (a -> c) -> (b -> d) -> f a b -> f c d
```

`bimap` can be thought of as applying a pair of `fmap` operations to a datatype.

A correct instance of `Bifunctor` for a type `f` must satisfy the bifunctor laws, which are analogous to the functor laws:

```
bimap id id = id -- identity
bimap (f . g) (h . i) = bimap f h . bimap g i -- composition
```

The Bifunctor class is found in the Data.Bifunctor module. For GHC versions >7.10, this module is bundled with the compiler; for earlier versions you need to install the bifunctors package.

Read Bifunctor online: https://riptutorial.com/haskell/topic/8020/bifunctor
Chapter 8: Cabal

Syntax

- cabal <command> where <command> is one of:
  - [global]
    - update
    - install
    - help
    - info
    - list
    - fetch
    - user-config
  - [package]
    - get
    - init
    - configure
    - build
    - clean
    - run
    - repl
    - test
    - bench
    - check
    - sdist
    - upload

https://riptutorial.com/
- Uploads source packages or documentation to Hackage
  - report
    - Upload build reports to a remote server
  - freeze
    - Freeze dependencies
  - gen-bounds
    - Generate dependency bounds
  - haddock
    - Generate Haddock HTML documentation
  - hscolour
    - Generate HsColour colourised code, in HTML format
  - copy
    - Copy the files into the install locations
  - register
    - Register this package with the compiler
- **sandbox**
  - sandbox
    - Create/modify/delete a sandbox
      - cabal sandbox init [FLAGS]
      - cabal sandbox delete [FLAGS]
      - cabal sandbox add-source [FLAGS] PATHS
      - cabal sandbox delete-source [FLAGS] PATHS
      - cabal sandbox list-sources [FLAGS]
      - cabal sandbox hc-pkg [FLAGS] [-] COMMAND [-] [ARGS]
  - exec
    - Give a command access to the sandbox package repository
  - repl
    - Open interpreter with access to sandbox packages

## Examples

### Install packages

To install a new package, e.g. aeson:

```
cabal install aeson
```

### Working with sandboxes

A Haskell project can either use the system wide packages or use a sandbox. A sandbox is an isolated package database and can prevent dependency conflicts, e.g. if multiple Haskell projects use different versions of a package.

To initialize a sandbox for a Haskell package go to its directory and run:

```
cabal sandbox init
```
Now packages can be installed by simply running `cabal install`.

Listing packages in a sandbox:
```
cabal sandbox hc-pkg list
```

Deleting a sandbox:
```
cabal sandbox delete
```

Add local dependency:
```
cabal sandbox add-source /path/to/dependency
```

Read Cabal online: https://riptutorial.com/haskell/topic/4740/cabal
Chapter 9: Category Theory

Examples

Category theory as a system for organizing abstraction

Category theory is a modern mathematical theory and a branch of abstract algebra focused on the nature of connectedness and relation. It is useful for giving solid foundations and common language to many highly reusable programming abstractions. Haskell uses Category theory as inspiration for some of the core typeclasses available in both the standard library and several popular third-party libraries.

An example

The Functo typeclass says that if a type $F$ instantiates Functor (for which we write $\text{Functor } F$) then we have a generic operation

$$\text{fmap :: (a } \rightarrow \text{ b) } \rightarrow \text{ (F a } \rightarrow \text{ F b)}$$

which lets us "map" over $F$. The standard (but imperfect) intuition is that $F \text{ a}$ is a container full of values of type $a$ and $\text{fmap}$ lets us apply a transformation to each of these contained elements. An example is Maybe

```haskell
instance Functor Maybe where
  fmap f Nothing = Nothing -- if there are no values contained, do nothing
  fmap f (Just a) = Just (f a) -- else, apply our transformation
```

Given this intuition, a common question is "why not call Functor something obvious like Mappable?".

A hint of Category Theory

The reason is that Functor fits into a set of common structures in Category theory and therefore by calling Functor "Functor" we can see how it connects to this deeper body of knowledge.

In particular, Category Theory is highly concerned with the idea of arrows from one place to another. In Haskell, the most important set of arrows are the function arrows $a \rightarrow b$. A common thing to study in Category Theory is how one set of arrows relates to another set. In particular, for any type constructor $F$, the set of arrows of the shape $F \text{ a } \rightarrow \text{ F b}$ are also interesting.

So a Functor is any $F$ such that there is a connection between normal Haskell arrows $a \rightarrow b$ and the $F$-specific arrows $F \text{ a } \rightarrow \text{ F b}$. The connection is defined by $\text{fmap}$ and we also recognize a few laws which must hold.
forall (x :: F a) . fmap id x == x
forall (f :: a -> b) (g :: b -> c) . fmap g . fmap f = fmap (g . f)

All of these laws arise naturally from the Category Theoretic interpretation of Functor and would not be as obviously necessary if we only thought of Functor as relating to "mapping over elements".

**Definition of a Category**

A category $C$ consists of:

- A collection of objects called $\text{Obj}(C)$;
- A collection (called $\text{Hom}(C)$) of morphisms between those objects. If $a$ and $b$ are in $\text{Obj}(C)$, then a morphism $f$ in $\text{Hom}(C)$ is typically denoted $f : a \to b$, and the collection of all morphism between $a$ and $b$ is denoted $\text{hom}(a,b)$;
- A special morphism called the identity morphism - for every $a : \text{Obj}(C)$ there exists a morphism $\text{id} : a \to a$;
- A composition operator $(.)$, taking two morphisms $f : a \to b, g : b \to c$ and producing a morphism $a \to c$

which obey the following laws:

For all $f : a \to x, g : x \to b$, then $\text{id} \cdot f = f$ and $g \cdot \text{id} = g$

For all $f : a \to b, g : b \to c$ and $h : c \to d$, then $h \cdot (g \cdot f) = (h \cdot g) \cdot f$

In other words, composition with the identity morphism (on either the left or right) does not change the other morphism, and composition is associative.

In Haskell, the Category is defined as a typeclass in `Control.Category`:

```haskell
class Category cat where
    id :: cat a a
    (.) :: cat b c -> cat a b -> cat a c
```

In this case, $\text{cat} :: k -> k -> *$ objectifies the morphism relation - there exists a morphism $\text{cat} a b$ if and only if $\text{cat} a b$ is inhabited (i.e. has a value). $a, b$ and $c$ are all in $\text{Obj}(C)$. $\text{Obj}(C)$ itself is represented by the kind $k$ - for example, when $k \sim *$, as is typically the case, objects are types.

The canonical example of a Category in Haskell is the function category:

```haskell
instance Category (->) where
    id = Prelude.id
    (.) = Prelude.(
```

https://riptutorial.com/
Another common example is the **Category of Kleisli arrows for a Monad**:

```haskell
newtype Kleisli m a b = Kleisli (a -> m b)

class Monad m => Category (Kleisli m) where
    id = Kleisli return
    Kleisli f . Kleisli g = Kleisli (f >>= g)
```

### Haskell types as a category

#### Definition of the category

The Haskell types along with functions between types form (almost†) a category. We have an identity morphism (function) \( \text{id} :: a \rightarrow a \) for every object (type) \( a \); and composition of morphisms \( (\cdot) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c \), which obey category laws:

\[
\begin{align*}
    f \cdot \text{id} &= f = \text{id} \cdot f \\
    h \cdot (g \cdot f) &= (h \cdot g) \cdot f
\end{align*}
\]

We usually call this category **Hask**.

#### Isomorphisms

In category theory, we have an isomorphism when we have a morphism which has an inverse, in other words, there is a morphism which can be composed with it in order to create the identity. In **Hask** this amounts to have a pair of morphisms \( f, g \) such that:

\[
\begin{align*}
    f \cdot g &= \text{id} = g \cdot f
\end{align*}
\]

If we find a pair of such morphisms between two types, we call them **isomorphic to one another**.

An example of two isomorphic types would be \(((,),a)\) and \(a\) for some \(a\). We can construct the two morphisms:

\[
\begin{align*}
    f :: ((,),a) \rightarrow a \\
    f ((,),x) &= x \\
    g :: a \rightarrow ((,),a) \\
    g x &= ((,),x)
\end{align*}
\]

And we can check that \(f \cdot g = \text{id} = g \cdot f\).

#### Functors

A functor, in category theory, goes from a category to another, mapping objects and morphisms. We are working only on one category, the category **Hask** of Haskell types, so we are going to see only functors from **Hask** to **Hask**, those functors, whose origin and destination category are the
same, are called **endofunctors**. Our endofunctors will be the polymorphic types taking a type and returning another:

\[
F :: * \to *
\]

To obey the categorical functor laws (preserve identities and composition) is equivalent to obey the Haskell functor laws:

\[
\begin{align*}
\text{fmap} \ (f \ . \ g) & = (\text{fmap} \ f) \ . \ (\text{fmap} \ g) \\
\text{fmap} \ \text{id} & = \text{id}
\end{align*}
\]

So, we have, for example, that \([], \text{Maybe}\) or \((\to \ r)\) are functors in Hask.

**Monads**

A monad in category theory is a monoid on the category of endofunctors. This category has endofunctors as objects \(F :: * \to *\) and natural transformations (transformations between them for all \(a \cdot F \ a \to G \ a\)) as morphisms.

A monoid object can be defined on a monoidal category, and is a type having two morphisms:

\[
\begin{align*}
\text{zero} :: () \to M \\
\text{mappend} :: (M, M) \to M
\end{align*}
\]

We can translate this roughly to the category of Hask endofunctors as:

\[
\begin{align*}
\text{return} :: a \to m \ a \\
\text{join} :: m (m \ a) \to m \ a
\end{align*}
\]

And, to obey the monad laws is equivalent to obey the categorical monoid object laws.

†In fact, the class of all types along with the class of functions between types do not strictly form a category in Haskell, due to the existence of \text{undefined}. Typically this is remedied by simply defining the objects of the Hask category as types without bottom values, which excludes non-terminating functions and infinite values (codata). For a detailed discussion of this topic, see here.

**Product of types in Hask**

**Categorical products**

In category theory, the product of two objects \(X, Y\) is another object \(Z\) with two projections: \(\pi_1 : Z \to X\) and \(\pi_2 : Z \to Y\); such that any other two morphisms from another object decompose uniquely through those projections. In other words, if there exist \(f_1 : W \to X\) and \(f_2 : W \to Y\), exists a unique morphism \(g : W \to Z\) such that \(\pi_1 \circ g = f_1\) and \(\pi_2 \circ g = f_2\).
Products in Hask

This translates into the Hask category of Haskell types as follows, \( z \) is product of \( A, B \) when:

```
-- if there are two functions
f1 :: W -> A
f2 :: W -> B
-- we can construct a unique function
g :: W -> Z
-- and we have two projections
p1 :: Z -> A
p2 :: Z -> B
-- such that the other two functions decompose using g
p1 . g == f1
p2 . g == f2
```

The **product type of two types** \( A, B \), which follows the law stated above, **is the tuple** of the two types \( (A, B) \), and the two projections are \( \text{fst} \) and \( \text{snd} \). We can check that it follows the above rule, if we have two functions \( f1 :: W -> A \) and \( f2 :: W -> B \) we can decompose them uniquely as follow:

```
decompose :: (W -> A) -> (W -> B) -> (W -> (A,B))
decompose f1 f2 = (\x -> (f1 x, f2 x))
```

And we can check that the decomposition is correct:

```
fst . (decompose f1 f2) = f1
snd . (decompose f1 f2) = f2
```

**Uniqueness up to isomorphism**

The choice of \( (A, B) \) as the product of \( A \) and \( B \) is not unique. Another logical and equivalent choice would have been:

```
data Pair a b = Pair a b
```

Moreover, we could have also chosen \( (B, A) \) as the product, or even \( (B, A, ()) \), and we could find a decomposition function like the above also following the rules:

```
decompose2 :: (W -> A) -> (W -> B) -> (W -> (B,A,()))
decompose2 f1 f2 = (\x -> (f2 x, f1 x, ()))
```

This is because the product is not unique but **unique up to isomorphism**. Every two products of \( A \) and \( B \) do not have to be equal, but they should be isomorphic. As an example, the two different products we have just defined, \( (A, B) \) and \( (B, A, ()) \), are isomorphic:

```
is1 :: (A,B) -> (B,A,())
is1 (x,y) = (y,x,())

iso2 :: (B,A,()) -> (A,B)
```
Uniqueness of the decomposition

It is important to remark that also the decomposition function must be unique. There are types which follow all the rules required to be product, but the decomposition is not unique. As an example, we can try to use \((A, (B, \text{Bool}))\) with projections \(\text{fst} \ . \ \text{snd}\) as a product of \(A\) and \(B\):

\[
\text{decompose3} :: (W \to A) \to (W \to B) \to (W \to (A, (B, \text{Bool})))
\]

\[
\text{decompose3} \ f1 \ f2 = (\lambda x \to (f1 \ x, (f2 \ x, \text{True})))
\]

We can check that it does work:

\[
\text{fst} \ . \ (\text{decompose3} \ f1 \ f2) = f1 \ x
\]

\[
(\text{fst} \ . \ \text{snd}) \ . \ (\text{decompose3} \ f1 \ f2) = f2 \ x
\]

But the problem here is that we could have written another decomposition, namely:

\[
\text{decompose3'} :: (W \to A) \to (W \to B) \to (W \to (A, (B, \text{Bool})))
\]

\[
\text{decompose3'} \ f1 \ f2 = (\lambda x \to (f1 \ x, (f2 \ x, \text{False})))
\]

And, as the decomposition is not unique, \((A, (B, \text{Bool}))\) is not the product of \(A\) and \(B\) in Hask

Coproduct of types in Hask

Intuition

The categorical product of two types \(A\) and \(B\) should contain the minimal information necessary to contain inside an instance of type \(A\) or type \(B\). We can see now that the intuitive coproduct of two types should be \(\text{Either} \ a \ b\). Other candidates, such as \(\text{Either} \ a \ (b, \text{Bool})\), would contain a part of unnecessary information, and they wouldn't be minimal.

The formal definition is derived from the categorical definition of coproduct.

Categorical coproducts

A categorical coproduct is the dual notion of a categorical product. It is obtained directly by reversing all the arrows in the definition of the product. The coproduct of two objects \(X, Y\) is another object \(Z\) with two inclusions: \(i_1 : X \to Z\) and \(i_2 : Y \to Z\); such that any other two morphisms from \(X\) and \(Y\) to another object decompose uniquely through those inclusions. In other words, if there are two morphisms \(f_1 : X \to W\) and \(f_2 : Y \to W\), exists a unique morphism \(g : Z \to W\) such that \(g \circ i_1 = f_1\) and \(g \circ i_2 = f_2\)

Coproducts in Hask
The translation into the **Hask** category is similar to the translation of the product:

```haskell
-- if there are two functions
f1 :: A -> W
f2 :: B -> W
-- and we have a coproduct with two inclusions
i1 :: A -> Z
i2 :: B -> Z
-- we can construct a unique function
g :: Z -> W
-- such that the other two functions decompose using g
  g . i1 == f1
  g . i2 == f2
```

The coproduct type of two types **A** and **B** in **Hask** is **Either a b** or any other type isomorphic to it:

```haskell
-- Coproduct
-- The two inclusions are Left and Right
data Either a b = Left a | Right b
-- If we have those functions, we can decompose them through the coproduct
decompose :: (A -> W) -> (B -> W) -> (Either A B -> W)
decompose f1 f2 (Left x)  = f1 x
decompose f1 f2 (Right y) = f2 y
```

**Haskell Applicative in terms of Category Theory**

A Haskell's **Functor** allows one to map any type **a** (an object of **Hask**) to a type **F a** and also map a function **a -> b** (a morphism of **Hask**) to a function with type **F a -> F b**. This corresponds to a Category Theory definition in a sense that functor preserves basic category structure.

A **monoidal category** is a category that has some additional structure:

- A tensor product (see Product of types in **Hask**)
- A tensor unit (unit object)

Taking a pair as our product, this definition can be translated to Haskell in the following way:

```haskell
class Functor f => Monoidal f where
    mcat :: f a -> f b -> f (a,b)
    munit :: f ()
```

The **Applicative** class is equivalent to this **Monoidal** one and thus can be implemented in terms of it:

```haskell
instance Monoidal f => Applicative f where
    pure x = fmap (const x) munit
    f <*> fa = (\(f, a) -> f a) <$> (mcat f fa)
```

Read Category Theory online: [https://riptutorial.com/haskell/topic/2261/category-theory](https://riptutorial.com/haskell/topic/2261/category-theory)
Chapter 10: Common functors as the base of cofree comonads

Examples

Cofree Empty ~~ Empty

Given

```haskell
data Empty a
```
we have

```haskell
data Cofree Empty a
   -- = a :< ... not possible!
```

Cofree (Const c) ~~ Writer c

Given

```haskell
data Const c a = Const c
```
we have

```haskell
data Cofree (Const c) a
   = a :< Const c
```
which is isomorphic to

```haskell
data Writer c a = Writer c a
```

Cofree Identity ~~ Stream

Given

```haskell
data Identity a = Identity a
```
we have

```haskell
data Cofree Identity a
   = a :< Identity (Cofree Identity a)
```
which is isomorphic to

https://riptutorial.com/
data Stream a = Stream a (Stream a)

Cofree Maybe ~~ NonEmpty

Given

data Maybe a = Just a
  | Nothing

we have

data Cofree Maybe a  
  = a :< Just (Cofree Maybe a)
  | a :< Nothing

which is isomorphic to

data NonEmpty a  
  = NECons a (NonEmpty a)
  | NESingle a

Cofree (Writer w) ~~ WriterT w Stream

Given

data Writer w a = Writer w a

we have

data Cofree (Writer w) a  
  = a :< (w, Cofree (Writer w) a)

which is equivalent to

data Stream (w,a)  
  = Stream (w,a) (Stream (w,a))

which can properly be written as \texttt{WriterT w Stream} with

data WriterT w m a = WriterT (m (w,a))

Cofree (Either e) ~~ NonEmptyT (Writer e)

Given

data Either e a = Left e
  | Right a
we have

```haskell
data Cofree (Either e) a
  = a :< Left e
  | a :< Right (Cofree (Either e) a)
```

which is isomorphic to

```haskell
data Hospitable e a
  = Sorry_AllIHaveIsThis_Here'sWhy a e
  | EatThis a (Hospitable e a)
```
or, if you promise to only evaluate the log after the complete result, `NonEmptyT (Writer e) a` with

```haskell
data NonEmptyT (Writer e) a = NonEmptyT (e,a,[a])
```

**Cofree (Reader x) ~~ Moore x**

Given

```haskell
data Reader x a = Reader (x -> a)
```

we have

```haskell
data Cofree (Reader x) a
  = a :< (x -> Cofree (Reader x) a)
```

which is isomorphic to

```haskell
data Plant x a
  = Plant a (x -> Plant x a)
```

aka **Moore machine**.

Read Common functors as the base of cofree comonads online:

https://riptutorial.com/
Chapter 11: Common GHC Language Extensions

Remarks

These language extensions are typically available when using the Glasgow Haskell Compiler (GHC) as they are not part of the approved Haskell 2010 language Report. To use these extensions, one must either inform the compiler using a flag or place a LANGUAGE programa before the module keyword in a file. The official documentation can be found in section 7 of the GCH users guide.

The format of the LANGUAGE programa is {-# LANGUAGE ExtensionOne, ExtensionTwo ... #-}. That is the literal {-# followed by LANGUAGE followed by a comma separated list of extensions, and finally the closing #-}. Multiple LANGUAGE programas may be placed in one file.

Examples

MultiParamTypeClasses

It's a very common extension that allows type classes with multiple type parameters. You can think of MPTC as a relation between types.

```haskell
{-# LANGUAGE MultiParamTypeClasses #-}

class Convertable a b where
    convert :: a -> b

instance Convertable Int Float where
    convert i = fromIntegral i
```

The order of parameters matters.

MPTCs can sometimes be replaced with type families.

FlexibleInstances

Regular instances require:

```
All instance types must be of the form (T a1 ... an)
where a1 ... an are “distinct type variables”,
and each type variable appears at most once in the instance head.
```

That means that, for example, while you can create an instance for [a] you can't create an instance for specifically [Int].; FlexibleInstances relaxes that:

```haskell
class C a where
```
-- works out of the box
instance C [a] where

-- requires FlexibleInstances
instance C [Int] where

OverloadedStrings

Normally, string literals in Haskell have a type of `String` (which is a type alias for `[Char]`). While this isn't a problem for smaller, educational programs, real-world applications often require more efficient storage such as `Text` or `ByteString`.

OverloadedStrings simply changes the type of literals to

```haskell
"test" :: Data.String.IsString a => a
```

Allowing them to be directly passed to functions expecting such a type. Many libraries implement this interface for their string-like types including `Data.Text` and `Data.ByteString` which both provide certain time and space advantages over `[Char]`.

There are also some unique uses of OverloadedStrings like those from the Postgresql-simple library which allows SQL queries to be written in double quotes like a normal string, but provides protections against improper concatenation, a notorious source of SQL injection attacks.

To create a instance of the `IsString` class you need to implement the `fromString` function. Example†:

```haskell
data Foo = A | B | Other String deriving Show

instance IsString Foo where
  fromString "A" = A
  fromString "B" = B
  fromString xs  = Other xs

tests :: [ Foo ]
tests = [ "A", "B", "Testing" ]
```

† This example courtesy of Lyndon Maydwell (sordina on GitHub) found here.

TupleSections

A syntactic extension that allows applying the tuple constructor (which is an operator) in a section way:

```haskell
(a,b) == (,) a b

-- With TupleSections
(a,b) == (,) a b == (a,) b == (,b) a
```
**N-tuples**

It also works for tuples with arity greater than two

```
(,) 1 3 == (1,2,3)
```

---

**Mapping**

This can be useful in other places where sections are used:

```
map (,"tag") [1,2,3] == [(1,"tag"), (2, "tag"), (3, "tag")]
```

The above example without this extension would look like this:

```
map (\a -> (a, "tag")) [1,2,3]
```

---

**UnicodeSyntax**

An extension that allows you to use Unicode characters in lieu of certain built-in operators and names.

<table>
<thead>
<tr>
<th>ASCII</th>
<th>Unicode</th>
<th>Use(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>::</td>
<td>::</td>
<td>has type</td>
</tr>
<tr>
<td>-&gt;</td>
<td>→</td>
<td>function types, lambdas, case branches, etc.</td>
</tr>
<tr>
<td>=&gt;</td>
<td>→</td>
<td>class constraints</td>
</tr>
<tr>
<td>forall</td>
<td>∀</td>
<td>explicit polymorphism</td>
</tr>
<tr>
<td>&lt;-</td>
<td>←</td>
<td>do notation</td>
</tr>
<tr>
<td>*</td>
<td>★</td>
<td>the type (or kind) of types (e.g., Int :: ★)</td>
</tr>
<tr>
<td>&gt;=</td>
<td>⩾</td>
<td>proc notation for Arrows</td>
</tr>
<tr>
<td>&lt;=</td>
<td>⩽</td>
<td>proc notation for Arrows</td>
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<td>&gt;&gt;-</td>
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<tr>
<td>&lt;&lt;&lt;</td>
<td>proc notation for Arrows</td>
<td></td>
</tr>
</tbody>
</table>

For example:
runST :: (forall s. ST s a) -> a

would become

runST :: (∀ s. ST s a) → a

Note that the * vs. ★ example is slightly different: since * isn’t reserved, ★ also works the same way as * for multiplication, or any other function named (★), and vice-versa. For example:

ghci> 2 ★ 3
6
ghci> let (*) = (+) in 2 ★ 3
5
ghci> let (★) = (-) in 2 * 3
-1

BinaryLiterals

Standard Haskell allows you to write integer literals in decimal (without any prefix), hexadecimal (preceded by 0x or 0X), and octal (preceded by 0o or 0O). The BinaryLiterals extension adds the option of binary (preceded by 0b or 0B).

0b1111 == 15     -- evaluates to: True

ExistentialQuantification

This is a type system extension that allows types that are existentially quantified, or, in other words, have type variables that only get instantiated at runtime†.

A value of existential type is similar to an abstract-base-class reference in OO languages: you don’t know the exact type in contains, but you can constrain the class of types.

data S = forall a. Show a => S a

or equivalently, with GADT syntax:

{-# LANGUAGE GADTs #-}
data S where
  S :: Show a => a -> S

Existential types open the door to things like almost-heterogenous containers: as said above, there can actually be various types in an S value, but all of them can be shown, hence you can also do

instance Show S where
  show (S a) = show a      -- we rely on (Show a) from the above
Now we can create a collection of such objects:

```haskell
ss = [S 5, S "test", S 3.0]
```

Which also allows us to use the polymorphic behaviour:

```haskell
mapM_ print ss
```

Existentials can be very powerful, but note that they are actually not necessary very often in Haskell. In the example above, all you can actually do with the `Show` instance is show (duh!) the values, i.e. create a string representation. The entire `S` type therefore contains exactly as much information as the string you get when showing it. Therefore, it is usually better to simply store that string right away, especially since Haskell is lazy and therefore the string will at first only be an unevaluated thunk anyway.

On the other hand, existentials cause some unique problems. For instance, the way the type information is “hidden” in an existential. If you pattern-match on an `S` value, you will have the contained type in scope (more precisely, its `Show` instance), but this information can never escape its scope, which therefore becomes a bit of a “secret society”: the compiler doesn’t let anything escape the scope except values whose type is already known from the outside. This can lead to strange errors like `Couldn't match type `a0' with `()' `a0' is untouchable.`

Contrast this with ordinary parametric polymorphism, which is generally resolved at compile time (allowing full type erasure).

Existential types are different from Rank-N types – these extensions are, roughly speaking, dual to each other: to actually use values of an existential type, you need a (possibly constrained-) polymorphic function, like `show` in the example. A polymorphic function is universally quantified, i.e. it works for any type in a given class, whereas existential quantification means it works for some particular type which is a priori unknown. If you have a polymorphic function, that’s sufficient, however to pass polymorphic functions as such as arguments, you need `{-# LANGUAGE Rank2Types #-}`:

```haskell
genShowSs :: (∀ x . Show x => x -> String) -> [S] -> [String]
genShowSs f = map (\(S a) -> f a)
```

**LambdaCase**

A syntactic extension that lets you write `\case` in place of `\arg -> case arg of`.

Consider the following function definition:

```haskell
dayOfTheWeek :: Int -> String
dayOfTheWeek 0 = "Sunday"
dayOfTheWeek 1 = "Monday"
dayOfTheWeek 2 = "Tuesday"
dayOfTheWeek 3 = "Wednesday"
```

https://riptutorial.com/
dayOfTheWeek 4 = "Thursday"
dayOfTheWeek 5 = "Friday"
dayOfTheWeek 6 = "Saturday"

If you want to avoid repeating the function name, you might write something like:

dayOfTheWeek :: Int -> String
dayOfTheWeek i = case i of
  0 -> "Sunday"
  1 -> "Monday"
  2 -> "Tuesday"
  3 -> "Wednesday"
  4 -> "Thursday"
  5 -> "Friday"
  6 -> "Saturday"

Using the LambdaCase extension, you can write that as a function expression, without having to name the argument:

{-# LANGUAGE LambdaCase #-}
dayOfTheWeek :: Int -> String
dayOfTheWeek = \case
  0 -> "Sunday"
  1 -> "Monday"
  2 -> "Tuesday"
  3 -> "Wednesday"
  4 -> "Thursday"
  5 -> "Friday"
  6 -> "Saturday"

RankNTypes

Imagine the following situation:

foo :: Show a => (a -> String) -> String -> Int -> IO ()
foo show' string int = do
  putStrLn (show' string)
  putStrLn (show' int)

Here, we want to pass in a function that converts a value into a String, apply that function to both a string parameter and an int parameter and print them both. In my mind, there is no reason this should fail! We have a function that works on both types of the parameters we're passing in.

Unfortunately, this won't type check! GHC infers the \(a\) type based off of its first occurrence in the function body. That is, as soon as we hit:

  putStrLn (show' string)

GHC will infer that \(\text{show}' :: \text{String} \to \text{String}\), since \(\text{string}\) is a \(\text{String}\). It will proceed to blow up while trying to \(\text{show}' \text{int}\).

RankNTypes
lets you instead write the type signature as follows, quantifying over all functions that satisfy the `show` type:

```haskell
foo :: (forall a. Show a => (a -> String)) -> String -> Int -> IO ()
```

This is rank 2 polymorphism: We are asserting that the `show` function must work for all \( a \) as within our function, and the previous implementation now works.

The `RankNTypes` extension allows arbitrary nesting of `forall` ... blocks in type signatures. In other words, it allows rank N polymorphism.

**OverloadedLists**

*added in GHC 7.8.*

OverloadedLists, similar to `OverloadedStrings`, allows list literals to be desugared as follows:

```haskell
[]          -- fromListN 0 []
[x]         -- fromListN 1 (x : [])
[x .. ]     -- fromList (enumFrom x)
```

This comes handy when dealing with types such as `Set`, `Vector` and `Map`s.

- `['0' .. '9']`          :: Set Char
- `[1 .. 10]`              :: Vector Int
- `["default",0], (k1,v1)` :: Map String Int
- `['a' .. 'z']`          :: Text

The `IsList` class in `GHC.Exts` is intended to be used with this extension.

`IsList` is equipped with one type function, `Item`, and three functions, `fromList :: [Item l] -> l`, `toList :: l -> [Item l]` and `fromListN :: Int -> [Item l] -> l` where `fromListN` is optional. Typical implementations are:

```haskell
instance IsList [a] where
  type Item [a] = a
  fromList = id
  toList = id

instance (Ord a) => IsList (Set a) where
  type Item (Set a) = a
  fromList = Set.fromList
  toList = Set.toList
```

*Examples taken from OverloadedLists – GHC.*

**FunctionalDependencies**

If you have a multi-parameter type-class with arguments \( a, b, c, \) and \( x \), this extension lets you express that the type \( x \) can be uniquely identified from \( a, b, \) and \( c \):

```haskell
https://riptutorial.com/  47
```
When declaring an instance of such class, it will be checked against all other instances to make sure that the functional dependency holds, that is, no other instance with same \( a b c \) but different \( x \) exists.

You can specify multiple dependencies in a comma-separated list:

```haskell
class OtherClass a b c d | a b -> c d, a d -> b where ...
```

For example in MTL we can see:

```haskell
class MonadReader r m | m -> r where ...
instance MonadReader r ((->) r) where ...
```

Now, if you have a value of type \( \text{MonadReader} a ((->) \text{Foo}) \rightarrow a \), the compiler can infer that \( a \sim \text{Foo} \), since the second argument completely determines the first, and will simplify the type accordingly.

The \( \text{SomeClass} \) class can be thought of as a function of the arguments \( a b c \) that results in \( x \). Such classes can be used to do computations in the typesystem.

**GADTs**

Conventional algebraic datatypes are parametric in their type variables. For example, if we define an ADT like

```haskell
data Expr a = IntLit Int
            | BoolLit Bool
            | If (Expr Bool) (Expr a) (Expr a)
```

with the hope that this will statically rule out non-well-typed conditionals, this will not behave as expected since the type of \( \text{IntLit} :: \text{Int} \rightarrow \text{Expr} \ a \) is universally quantified: for any choice of \( a \), it produces a value of type \( \text{Expr} \ a \). In particular, for \( a \sim \text{Bool} \), we have \( \text{IntLit} :: \text{Int} \rightarrow \text{Expr} \ \text{Bool} \), allowing us to construct something like \( \text{If} \ \text{IntLit} \ 1 \ \text{e1} \ \text{e2} \) which is what the type of the \( \text{If} \) constructor was trying to rule out.

Generalised Algebraic Data Types allows us to control the resulting type of a data constructor so that they are not merely parametric. We can rewrite our \( \text{Expr} \) type as a GADT like this:

```haskell
data Expr a where
  IntLit :: Int -> Expr Int
  BoolLit :: Bool -> Expr Bool
  If :: Expr Bool -> Expr a -> Expr a -> Expr a
```

Here, the type of the constructor \( \text{IntLit is Int} \rightarrow \text{Expr Int} \), and so \( \text{IntLit} \ 1 :: \text{Expr Bool} \) will not typecheck.

Pattern matching on a GADT value causes refinement of the type of the term returned. For example, it is possible to write an evaluator for \( \text{Expr} \ a \) like this:
crazyEval :: Expr a -> a
crazyEval (IntLit x) =
  -- Here we can use `(+)` because x :: Int
  x + 1
crazyEval (BoolLit b) =
  -- Here we can use `not` because b :: Bool
  not b
crazyEval (If b thn els) =
  -- Because b :: Expr Bool, we can use `crazyEval b :: Bool`.
  -- Also, because thn :: Expr a and els :: Expr a, we can pass either to
  -- the recursive call to `crazyEval` and get an a back
  crazyEval $ if crazyEval b then thn else els

Note that we are able to use `(+)` in the above definitions because when e.g. `IntLit x` is pattern matched, we also learn that `a ~ Int` (and likewise for `not` and `if_then_else_ when a ~ Bool`).

ScopedTypeVariables

ScopedTypeVariables let you refer to universally quantified types inside of a declaration. To be more explicit:

```haskell
import Data.Monoid

foo :: forall a b c. (Monoid b, Monoid c) => (a, b, c) -> (b, c) -> (a, b, c)
foo (a, b, c) (b', c') = (a :: a, b'', c'')
  where (b'', c'') = (b <> b', c <> c') :: (b, c)
```

The important thing is that we can use `a`, `b` and `c` to instruct the compiler in subexpressions of the declaration (the tuple in the `where` clause and the first `a` in the final result). In practice,ScopedTypeVariables assist in writing complex functions as a sum of parts, allowing the programmer to add type signatures to intermediate values that don’t have concrete types.

PatternSynonyms

Pattern synonyms are abstractions of patterns similar to how functions are abstractions of expressions.

For this example, let's look at the interface `Data.Sequence` exposes, and let's see how it can be improved with pattern synonyms. The `Seq` type is a data type that, internally, uses a complicated representation to achieve good asymptotic complexity for various operations, most notably both \(O(1)\) (un)consing and (un)snocing.

But this representation is unwieldy and some of its invariants cannot be expressed in Haskell's type system. Because of this, the `Seq` type is exposed to users as an abstract type, along with invariant-preserving accessor and constructor functions, among them:

```haskell
empty :: Seq a

(<|) :: a -> Seq a -> Seq a
data ViewL a = EmptyL | a :< (Seq a)
viewl :: Seq a -> ViewL a
```
(|>) :: Seq a -> a -> Seq a

data ViewR a = EmptyR | (Seq a) :> a

viewr :: Seq a -> ViewR a

But using this interface can be a bit cumbersome:

uncons :: Seq a -> Maybe (a, Seq a)
uncons xs = case viewl xs of
  x :< xs' -> Just (x, xs')
  EmptyL -> Nothing

We can use view patterns to clean it up somewhat:

{-# LANGUAGE ViewPatterns #-}

uncons :: Seq a -> Maybe (a, Seq a)
uncons (viewl -> x :< xs) = Just (x, xs)
uncons _ = Nothing

Using the PatternSynonyms language extension, we can give an even nicer interface by allowing pattern matching to pretend that we have a cons- or a snoc-list:

{-# LANGUAGE PatternSynonyms #-}

import Data.Sequence (Seq)
import qualified Data.Sequence as Seq

pattern Empty :: Seq a
pattern Empty <- (Seq.viewl -> Seq.EmptyL)

pattern (:<) :: a -> Seq a -> Seq a
pattern x :< xs <- (Seq.viewl -> x Seq.<: xs)

pattern (:>) :: Seq a -> a -> Seq a
pattern xs :> x <- (Seq.viewr -> xs Seq.>: x)

This allows us to write uncons in a very natural style:

uncons :: Seq a -> Maybe (a, Seq a)
uncons (x :< xs) = Just (x, xs)
uncons _ = Nothing

RecordWildCards

See RecordWildCards

Read Common GHC Language Extensions online:
https://riptutorial.com/haskell/topic/1274/common-ghc-language-extensions
Chapter 12: Common monads as free monads

Examples

Free Empty ~~ Identity

Given

data Empty a

we have

data Free Empty a
= Pure a
-- the Free constructor is impossible!

which is isomorphic to

data Identity a
= Identity a

Free Identity ~~ (Nat,) ~~ Writer Nat

Given

data Identity a = Identity a

we have

data Free Identity a
= Pure a
  | Free (Identity (Free Identity a))

which is isomorphic to

data Deferred a
= Now a
  | Later (Deferred a)

or equivalently (if you promise to evaluate the fst element first) \((\text{Nat}, a)\), aka \(\text{Writer Nat a}\), with

data Nat = Z | S Nat
data Writer Nat a = Writer Nat a
Free Maybe ~~ MaybeT (Writer Nat)

Given

data Maybe a = Just a  
| Nothing

we have

data Free Maybe a
    = Pure a
    | Free (Just (Free Maybe a))
    | Free Nothing

which is equivalent to

data Hopes a
    = Confirmed a
    | Possible (Hopes a)
    | Failed

or equivalently (if you promise to evaluate the fst element first) (Nat, Maybe a), aka MaybeT (Writer Nat) a with

data Nat = Z | S Nat  
data Writer Nat a = Writer Nat a  
data MaybeT (Writer Nat) a = MaybeT (Nat, Maybe a)

Free (Writer w) ~~ Writer [w]

Given

data Writer w a = Writer w a

we have

data Free (Writer w) a
    = Pure a
    | Free (Writer w (Free (Writer w) a))

which is isomorphic to

data ProgLog w a
    = Done a
    | After w (ProgLog w a)

or, equivalently, (if you promise to evaluate the log first), Writer [w] a.

Free (Const c) ~~ Either c
Given

data Const c a = Const c

we have

data Free (Const c) a
  = Pure a
  | Free (Const c)

which is isomorphic to

data Either c a
  = Right a
  | Left c

Free (Reader x) ~~ Reader (Stream x)

Given

data Reader x a = Reader (x -> a)

we have

data Free (Reader x) a
  = Pure a
  | Free (x -> Free (Reader x) a)

which is isomorphic to

data Demand x a
  = Satisfied a
  | Hungry (x -> Demand x a)

or equivalently Stream x -> a with

data Stream x = Stream x (Stream x)

Read Common monads as free monads online: https://riptutorial.com/haskell/topic/8256/common-monads-as-free-monads
Chapter 13: Concurrency

Remarks

Good resources for learning about concurrent and parallel programming in Haskell are:

- Parallel and Concurrent Programming in Haskell
- the Haskell Wiki

Examples

Spawning Threads with `forkIO`

Haskell supports many forms of concurrency and the most obvious being forking a thread using forkIO.

The function `forkIO :: IO () -> IO ThreadId` takes an IO action and returns its ThreadId, meanwhile the action will be run in the background.

We can demonstrate this quite succinctly using ghci:

```
Prelude Control.Concurrent> forkIO $ (print . sum) [1..100000000]  
ThreadId 290  
Prelude Control.Concurrent> forkIO $ print "hi!"  
"hi!"  
-- some time later....  
Prelude Control.Concurrent> 500000050000000
```

Both actions will run in the background, and the second is almost guaranteed to finish before the last!

Communicating between Threads with `MVar`

It is very easy to pass information between threads using the `MVar a` type and its accompanying functions in `Control.Concurrent`:

- `newEmptyMVar :: IO (MVar a)` --- creates a new `MVar a`
- `newMVar :: a -> IO (MVar a)` --- creates a new `MVar` with the given value
- `takeMVar :: MVar a -> IO a` --- retrieves the value from the given `MVar`, or `blocks` until one is available
- `putMVar :: MVar a -> a -> IO ()` --- puts the given value in the `MVar`, or `blocks` until it's empty

Let's sum the numbers from 1 to 100 million in a thread and wait on the result:

```
import Control.Concurrent
main = do
    import Control.Concurrent
    main = do
```

https://riptutorial.com/
m <- newEmptyMVar
forkIO $ putMVar m $ sum [1..1000000]
print =<< takeMVar m  -- takeMVar will block 'til m is non-empty!

A more complex demonstration might be to take user input and sum in the background while waiting for more input:

main2 = loop
  where
    loop = do
      m <- newEmptyMVar
      n <- getLine
      putStrLn "Calculating. Please wait"
      -- In another thread, parse the user input and sum
      forkIO $ putMVar m $ sum [1..(read n :: Int)]
      -- In another thread, wait 'til the sum's complete then print it
      forkIO $ print =<< takeMVar m
    loop

As stated earlier, if you call takeMVar and the MVar is empty, it blocks until another thread puts something into the MVar, which could result in a Dining Philosophers Problem. The same thing happens with putMVar: if it's full, it'll block 'til it's empty!

Take the following function:

concurrent ma mb = do
  a <- takeMVar ma
  b <- takeMVar mb
  putMVar ma a
  putMVar mb b

We run the the two functions with some MVars

concurrent ma mb     -- new thread 1
concurrent mb ma     -- new thread 2

What could happen is that:

1. Thread 1 reads ma and blocks ma
2. Thread 2 reads mb and thus blocks mb

Now Thread 1 cannot read mb as Thread 2 has blocked it, and Thread 2 cannot read ma as Thread 1 has blocked it. A classic deadlock!

Atomic Blocks with Software Transactional Memory

Another powerful & mature concurrency tool in Haskell is Software Transactional Memory, which allows for multiple threads to write to a single variable of type TVar a in an atomic manner.

TVar a is the main type associated with the STM monad and stands for transactional variable. They're used much like MVar but within the STM monad through the following functions:
atomically :: STM a -> IO a

Perform a series of STM actions atomically.

readTVar :: TVar a -> STM a

Read the `TVar`'s value, e.g.:

value <- readTVar t

writeTVar :: TVar a -> a -> STM ()

Write a value to the given `TVar`.

t <- newTVar Nothing
writeTVar t (Just "Hello")

This example is taken from the Haskell Wiki:

```haskell
import Control.Monad
import Control.Concurrent
import Control.Concurrent.STM

main = do
    -- Initialise a new `TVar`
    shared <- atomically $ newTVar 0
    -- Read the value
    before <- atomRead shared
    putStrLn $ "Before: " ++ show before
    forkIO $ 25 `timesDo` (dispVar shared >> milliSleep 20)
    forkIO $ 10 `timesDo` (appV (++) 2 shared >> milliSleep 50)
    forkIO $ 20 `timesDo` (appV pred shared >> milliSleep 25)
    milliSleep 800
    after <- atomRead shared
    putStrLn $ "After: " ++ show after
    where timesDo = replicateM_
        milliSleep = threadDelay . (*) 1000

    atomRead = atomically . readTVar
    dispVar x = atomRead x >>= print
    appV fn x = atomically $ readTVar x >>= writeTVar x . fn
```

Read Concurrency online: https://riptutorial.com/haskell/topic/4426/concurrency
Chapter 14: Containers - Data.Map

Examples

Constructing

We can create a Map from a list of tuples like this:

Map.fromList ["Alex", 31], ("Bob", 22)]

A Map can also be constructed with a single value:

> Map.singleton "Alex" 31
fromList ["Alex",31]]

There is also the empty function.

empty :: Map k a

Data.Map also supports typical set operations such as union, difference and intersection.

Checking If Empty

We use the null function to check if a given Map is empty:

> Map.null $ Map.fromList ["Alex", 31], ("Bob", 22)]
False
> Map.null $ Map.empty
True

Finding Values

There are many querying operations on maps.

member :: Ord k => k -> Map k a -> Bool yields True if the key of type k is in Map k a:

> Map.member "Alex" $ Map.singleton "Alex" 31
True
> Map.member "Jenny" $ Map.empty
False

notMember is similar:

> Map.notMember "Alex" $ Map.singleton "Alex" 31
False
> Map.notMember "Jenny" $ Map.empty
You can also use `findWithDefault :: Ord k => a -> k -> Map k a -> a` to yield a default value if the key isn't present:

Map.findWithDefault 'x' 1 (fromList [(5,'a'), (3,'b')]) == 'x'
Map.findWithDefault 'x' 5 (fromList [(5,'a'), (3,'b')]) == 'a'

### Inserting Elements

**Inserting** elements is simple:

```haskell
> let m = Map.singleton "Alex" 31
fromList [("Alex",31)]

> Map.insert "Bob" 99 m
fromList [("Alex",31),("Bob",99)]
```

### Deleting Elements

```haskell
> let m = Map.fromList [("Alex", 31), ("Bob", 99)]
fromList [("Alex",31),("Bob",99)]

> Map.delete "Bob" m
fromList [("Alex",31)]
```

### Importing the Module

The `Data.Map` module in the `containers` package provides a `Map` structure that has both strict and lazy implementations.

When using `Data.Map`, one usually imports it qualified to avoid clashes with functions already defined in Prelude:

```haskell
import qualified Data.Map as Map
```

So we’d then prepend `Map` function calls with `Map.`, e.g.

```haskell
Map.empty -- give me an empty Map
```

### Monoid instance

`Map k v` provides a `Monoid` instance with the following semantics:

- `mempty` is the empty `Map`, i.e. the same as `Map.empty`
- `m1 <> m2` is the left-biased union of `m1` and `m2`, i.e. if any key is present both in `m1` and `m2`, then the value from `m1` is picked for `m1 <> m2`. This operation is also available outside the `Monoid` instance as `Map.union`.

https://riptutorial.com/
Read Containers - Data.Map online: https://riptutorial.com/haskell/topic/4591/containers---data-map
Chapter 15: Creating Custom Data Types

Examples

Creating a simple data type

The easiest way to create a custom data type in Haskell is to use the `data` keyword:

```haskell
data Foo = Bar | Biz
```

The name of the type is specified between `data` and `=`, and is called a type constructor. After `=` we specify all value constructors of our data type, delimited by the `|` sign. There is a rule in Haskell that all type and value constructors must begin with a capital letter. The above declaration can be read as follows:

Define a type called `Foo`, which has two possible values: `Bar` and `Biz`.

Creating variables of our custom type

```haskell
let x = Bar
```

The above statement creates a variable named `x` of type `Foo`. Let's verify this by checking its type.

```haskell
:t x
```

prints

```
x :: Foo
```

Creating a data type with value constructor parameters

Value constructors are functions that return a value of a data type. Because of this, just like any other function, they can take one or more parameters:

```haskell
data Foo = Bar String Int | Biz String
```

Let's check the type of the `Bar` value constructor.

```haskell
:t Bar
```

prints

```
Bar :: String -> Int -> Foo
```
which proves that \( \text{Bar} \) is indeed a function.

**Creating variables of our custom type**

```haskell
let x = Bar "Hello" 10
let y = Biz "Goodbye"
```

**Creating a data type with type parameters**

Type constructors can take one or more type parameters:

```haskell
data Foo a b = Bar a b | Biz a b
```

Type parameters in Haskell must begin with a lowercase letter. Our custom data type is not a real type yet. In order to create values of our type, we must substitute all type parameters with actual types. Because \( a \) and \( b \) can be of any type, our value constructors are polymorphic functions.

**Creating variables of our custom type**

```haskell
let x = Bar "Hello" 10      -- x :: Foo [Char] Integer
let y = Biz "Goodbye" 6.0   -- y :: Fractional b => Foo [Char] b
let z = Biz True False      -- z :: Foo Bool Bool
```

**Custom data type with record parameters**

Assume we want to create a data type Person, which has a first and last name, an age, a phone number, a street, a zip code and a town.

We could write

```haskell
data Person = Person String String Int Int String String String
```

If we want now to get the phone number, we need to make a function

```haskell
getPhone :: Person -> Int
getPhone (Person _ _ _ phone _ _ _) = phone
```

Well, this is no fun. We can do better using parameters:

```haskell
data Person' = Person' { firstName :: String , lastName :: String , age :: Int , phone :: Int , street :: String , code :: String , town :: String }
```
Now we get the function `phone` where

\[
:t \text{ phone} \\
\text{phone :: Person' -> Int}
\]

We can now do whatever we want, eg:

\[
\text{printPhone :: Person' -> IO ()} \\
\text{printPhone = putStrLn . show . phone}
\]

We can also bind the phone number by **Pattern Matching**:

\[
\text{getPhone' :: Person' -> Int} \\
\text{getPhone' (Person \{ phone = p \}) = p}
\]

For easy use of the parameters see **RecordWildCards**

**Read Creating Custom Data Types online:** [https://riptutorial.com/haskell/topic/4057/creating-custom-data-types](https://riptutorial.com/haskell/topic/4057/creating-custom-data-types)
Chapter 16: Data.Aeson - JSON in Haskell

Examples

Smart Encoding and Decoding using Generics

The easiest and quickest way to encode a Haskell data type to JSON with Aeson is using generics.

```haskell
{-# LANGUAGE DeriveGeneric #-}
import GHC.Generics
import Data.Text
import Data.Aeson
import Data.ByteString.Lazy

First let us create a data type Person:

data Person = Person { firstName :: Text, lastName :: Text, age :: Int } deriving (Show, Generic)

In order to use the `encode` and `decode` function from the `Data.Aeson` package we need to make `Person` an instance of `ToJSON` and `FromJSON`. Since we derive `Generic` for `Person`, we can create empty instances for these classes. The default definitions of the methods are defined in terms of the methods provided by the `Generic` type class.

```haskell
instance ToJSON Person
instance FromJSON Person

Done! In order to improve the encoding speed we can slightly change the `ToJSON` instance:

```haskell
instance ToJSON Person where
toEncoding = genericToEncoding defaultOptions
```

Now we can use the `encode` function to convert `Person` to a (lazy) Bytestring:

```haskell
encodeNewPerson :: Text -> Text -> Int -> ByteString
encodeNewPerson first last age = encode $ Person first last age

> encodeNewPerson "Hans" "Wurst" 30
"{"lastName":"Wurst","age":30,"firstName":"Hans"}"

And to decode we can just use `decode`:

```haskell
> decode $ encodeNewPerson "Hans" "Wurst" 30
```

https://riptutorial.com/
Just (Person {firstName = "Hans", lastName = "Wurst", age = 30})

A quick way to generate a Data.Aeson.Value

{-# LANGUAGE OverloadedStrings #-}
module Main where

import Data.Aeson

main :: IO ()
main = do
  let example = Data.Aeson.object [ "key" .= (5 :: Integer), "somethingElse" .= (2 :: Integer) ] :: Value
      print . encode $ example

Optional Fields

Sometimes, we want some fields in the JSON string to be optional. For example,

data Person = Person { firstName :: Text,
                      , lastName  :: Text
                      , age       :: Maybe Int
                      }

This can be achieved by

import Data.Aeson.TH

$(deriveJSON defaultOptions{omitNothingFields = True} ''Person)

Chapter 17: Data.Text

Remarks

Text is a more efficient alternative to Haskell's standard String type. String is defined as a linked list of characters in the standard Prelude, per the Haskell Report:

```hs
type String = [Char]
```

Text is represented as a packed array of Unicode characters. This is similar to how most other high-level languages represent strings, and gives much better time and space efficiency than the list version.

Text should be preferred over String for all production usage. A notable exception is depending on a library which has a String API, but even in that case there may be a benefit of using Text internally and converting to a String just before interfacing with the library.

All of the examples in this topic use the OverloadedStrings language extension.

Examples

Text Literals

The OverloadedStrings language extension allows the use of normal string literals to stand for Text values.

```hs
{-# LANGUAGE OverloadedStrings #-}
import qualified Data.Text as T

myText :: T.Text
myText = "overloaded"
```

Stripping whitespace

```hs
{-# LANGUAGE OverloadedStrings #-}
import qualified Data.Text as T

myText :: T.Text
myText = "\n\r\t   leading and trailing whitespace   \t\r\n"

strip removes whitespace from the start and end of a Text value.

ghci> T.strip myText
"leading and trailing whitespace"
```
stripStart removes whitespace only from the start.

```ghci
T.stripStart myText
"leading and trailing whitespace  \t\r\n"
```

stripEnd removes whitespace only from the end.

```ghci
T.stripEnd myText
"\n\r\t  leading and trailing whitespace"
```

filter can be used to remove whitespace, or other characters, from the middle.

```ghci
T.filter /'= ' "spaces in the middle of a text string"
"spacesinthemiddleofatextstring"
```

Splitting Text Values

```haskell
{-# LANGUAGE OverloadedStrings #-}
import qualified Data.Text as T

myText :: T.Text
myText = "mississippi"
```

splitOn breaks a Text up into a list of Texts on occurrences of a substring.

```ghci
T.splitOn "ss" myText
["mi","i","ippi"]
```

splitOn is the inverse of intercalate.

```ghci
intercalate "ss" (splitOn "ss" "mississippi")
"mississippi"
```

split breaks a Text value into chunks on characters that satisfy a Boolean predicate.

```ghci
T.split (== 'i') myText
["m","ss","ss","pp","""]
```

Encoding and Decoding Text

Encoding and decoding functions for a variety of Unicode encodings can be found in the Data.Text.Encoding module.

```ghci
import Data.Text.Encoding
ghci> decodeUtf8 (encodeUtf8 "my text")
"my text"
```

Note that decodeUtf8 will throw an exception on invalid input. If you want to handle invalid UTF-8...
yourself, use `decodeUtf8With`.

```haskell
ghci> decodeUtf8With (\errorDescription input -> Nothing) messyOutsideData
```

### Checking if a Text is a substring of another Text

```haskell
ghci> :set -XOverloadedStrings
ghci> import Data.Text as T
```

- **`isInfixOf :: Text -> Text -> Bool`** checks whether a `Text` is contained anywhere within another `Text`.

  ```haskell
ghci> "rum" `T.isInfixOf` "crumble"  
  True
  ```

- **`isPrefixOf :: Text -> Text -> Bool`** checks whether a `Text` appears at the beginning of another `Text`.

  ```haskell
ghci> "crumb" `T.isPrefixOf` "crumble"  
  True
  ```

- **`isSuffixOf :: Text -> Text -> Bool`** checks whether a `Text` appears at the end of another `Text`.

  ```haskell
ghci> "rumble" `T.isSuffixOf` "crumble"  
  True
  ```

### Indexing Text

```haskell
{-# LANGUAGE OverloadedStrings #-}
import qualified Data.Text as T
myText :: T.Text
myText = "mississippi"
```

Characters at specific indices can be returned by the `index` function.

```haskell
ghci> T.index myText 2  
's'
```

The `findIndex` function takes a function of type `(Char -> Bool)` and `Text` and returns the index of the first occurrence of a given string or `Nothing` if it doesn't occur.

```haskell
ghci> T.findIndex ('s'==) myText  
Just 2
ghci> T.findIndex ('c'==) myText  
Nothing
```
The `count` function returns the number of times a query `Text` occurs within another `Text`.

```ghci
ghci> count ("miss"::T.Text) myText
1
```
Chapter 18: Databases

Examples

Postgres

Postgresql-simple is a mid-level Haskell library for communicating with a PostgreSQL backend database. It is very simple to use and provides a type-safe API for reading/writing to a DB.

Running a simple query is as easy as:

```haskell
{-# LANGUAGE OverloadedStrings #-}

import Database.PostgreSQL.Simple

main :: IO ()
main = do
  -- Connect using libpq strings
  conn <- connectPostgreSQL "host='my.dbhost' port=5432 user=bob pass=bob"
  [Only i] <- query_ conn "select 2 + 2" -- execute with no parameter substitution
  print i
```

Parameter substitution

PostgreSQL-Simple supports parameter substitution for safe parameterised queries using \texttt{query}:

```haskell
main :: IO ()
main = do
  -- Connect using libpq strings
  conn <- connectPostgreSQL "host='my.dbhost' port=5432 user=bob pass=bob"
  [Only i] <- query conn "select ? + ?" [1, 1]
  print i
```

Executing inserts or updates

You can run inserts/update SQL queries using \texttt{execute}:

```haskell
main :: IO ()
main = do
  -- Connect using libpq strings
  conn <- connectPostgreSQL "host='my.dbhost' port=5432 user=bob pass=bob"
  execute conn "insert into people (name, age) values (?, ?)" ["Alex", 31]
```

Read Databases online: https://riptutorial.com/haskell/topic/4444/databases
Chapter 19: Date and Time

Syntax

- `addDays :: Integer -> Day -> Day`
- `diffDays :: Day -> Day -> Integer`
- `fromGregorian :: Integer -> Int -> Int -> Day`

Converting from proleptic Gregorian calendar. First argument is year, second month number (1-12), third day (1-31). Invalid values will be clipped to the correct range, month first, then day.

- `getCurrentTime :: IO UTCTime`

Remarks

The `Data.Time` module from `time package` provides support for retrieving & manipulating date & time values:

Examples

Finding Today's Date

Current date and time can be found with `getCurrentTime`:

```haskell
import Data.Time

print =<< getCurrentTime
-- 2016-08-02 12:05:08.937169 UTC
```

Alternatively, just the date is returned by `fromGregorian`:

```haskell
fromGregorian 1984 11 17  -- yields a Day
```

Adding, Subtracting and Comparing Days

Given a `Day`, we can perform simple arithmetic and comparisons, such as adding:

```haskell
import Data.Time

addDays 1 (fromGregorian 2000 1 1)
-- 2000-01-02
addDays 1 (fromGregorian 2000 12 31)
-- 2001-01-01
```
Subtract:

\[ \text{addDays (-1) (fromGregorian 2000 1 1)} \]
\[ -- 1999-12-31 \]

\[ \text{addDays (-1) (fromGregorian 0 1 1)} \]
\[ -- -0001-12-31 \]
\[ -- \text{wat} \]

and even find the difference:

\[ \text{diffDays (fromGregorian 2000 12 31) (fromGregorian 2000 1 1)} \]
\[ 365 \]

note that the order matters:

\[ \text{diffDays (fromGregorian 2000 1 1) (fromGregorian 2000 12 31)} \]
\[ -365 \]

Read Date and Time online: https://riptutorial.com/haskell/topic/4950/date-and-time
Chapter 20: Fixity declarations

Syntax

1. infix [integer] ops
2. infixl [integer] ops
3. infixr [integer] ops

Parameters

<table>
<thead>
<tr>
<th>Declaration component</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>infixr</td>
<td>the operator is right-associative</td>
</tr>
<tr>
<td>infixl</td>
<td>the operator is left-associative</td>
</tr>
<tr>
<td>infix</td>
<td>the operator is non-associative</td>
</tr>
<tr>
<td>optional digit</td>
<td>binding precedence of the operator (range 0...9, default 9)</td>
</tr>
<tr>
<td>op1, ... , opn</td>
<td>operators</td>
</tr>
</tbody>
</table>

Remarks

To parse expressions involving operators and functions, Haskell uses fixity declarations to figure out where parenthesis go. In order, it

1. wraps function applications in parens
2. uses binding precedence to wrap groups of terms all seperated by operators of the same precedence
3. uses the associativity of those operators to figure out how to add parens to these groups

Notice that we assume here that the operators in any given group from step 2 must all have the same associativity. In fact, Haskell will reject any program where this condition is not met.

As an example of the above algorithm, we can step though the process of adding parenthesis to \(1 + \text{negate } 5 \ast 2 - 3 \ast 4 \wedge 2 \wedge 1\).

\[
\begin{align*}
\text{infixl 6 +} \\
\text{infixl 6 -} \\
\text{infixl 7 *} \\
\text{infixr 8 ^}
\end{align*}
\]

1. \(1 + (\text{negate } 5) \ast 2 - 3 \ast 4 \wedge 2 \wedge 1\)
2. \(1 + ((\text{negate } 5) \ast 2) - (3 \ast (4 \wedge (2 \wedge 1)))\)
3. \((1 + ((\text{negate } 5) \ast 2)) - (3 \ast (4 \wedge (2 \wedge 1)))\)
More details in section 4.4.2 of the Haskell 98 report.

Examples

Associativity

**infixl** vs **infixr** vs **infix** describe on which sides the parens will be grouped. For example, consider the following fixity declarations (in base)

```haskell
infixl 6 -
infixr 5 :
infix  4 ==
```

The **infixl** tells us that `-` has left associativity, which means that `1 - 2 - 3 - 4` gets parsed as

```haskell
((1 - 2) - 3) - 4
```

The **infixr** tells us that `:` has right associativity, which means that `1 : 2 : 3 : []` gets parsed as

```haskell
1 : (2 : (3 : []))
```

The **infix** tells us that `==` cannot be used without us including parenthesis, which means that `True == False == True` is a syntax error. On the other hand, `True == (False == True) or (True == False) == True` are fine.

Operators without an explicit fixity declaration are **infixl 9**.

Binding precedence

The number that follows the associativity information describes in what order the operators are applied. It must always be between 0 and 9 inclusive. This is commonly referred to as how tightly the operator binds. For example, consider the following fixity declarations (in base)

```haskell
infixl 6 +
infixl 7 *
```

Since `*` has a higher binding precedence than `+` we read `1 * 2 + 3` as

```haskell
(1 * 2) + 3
```

In short, the higher the number, the closer the operator will "pull" the parens on either side of it.

Remarks

- Function application *always* binds higher than operators, so `f x `op` g y` must be interpreted as `(f x)`op`(g y)` no matter what the operator `op` and its fixity declaration are.
• If the binding precedence is omitted in a fixity declaration (for example we have \texttt{infixl *!?}) the default is 9.

Example declarations

• \texttt{infixr 5 ++}
• \texttt{infixl 4 <*, *, >, <**>}
• \texttt{infixl 8 `shift`, `rotate`, `shiftL`, `shiftR`, `rotateL`, `rotateR`}
• \texttt{infix 4 ==, /=, <, <=, >=, >}
• \texttt{infix ??}

Read Fixity declarations online: \url{https://riptutorial.com/haskell/topic/4691/fixity-declarations}
Chapter 21: Foldable

Introduction

Foldable is the class of types \( t :: * \rightarrow * \) which admit a folding operation. A fold aggregates the elements of a structure in a well-defined order, using a combining function.

Remarks

If \( t \) is Foldable it means that for any value \( t \ a \) we know how to access all of the elements of \( a \) from "inside" of \( t \ a \) in a fixed linear order. This is the meaning of \( \text{foldMap} :: \text{Monoid } m \Rightarrow (a \rightarrow m) \rightarrow (t \ a \rightarrow m) \): we "visit" each element with a summary function and smash all the summaries together.

Monoids respect order (but are invariant to different groupings).

Examples

Counting the elements of a Foldable structure

\( \text{length} \) counts the occurrences of elements \( a \) in a foldable structure \( t \ a \).

```
ghci> length [7, 2, 9] -- t ~ []
3
ghci> length (Right 'a') -- t ~ Either e
1 -- 'Either e a' may contain zero or one 'a'
ghci> length (Left "foo") -- t ~ Either String
0
ghci> length (3, True) -- t ~ (,) Int
1 -- '(c, a)' always contains exactly one 'a'
```

\( \text{length} \) is defined as being equivalent to:

```
class Foldable t where
    -- ...
    length :: t a -> Int
    length = foldl' (\c _ -> c+1) 0
```

Note that this return type \( \text{Int} \) restricts the operations that can be performed on values obtained by calls to the \( \text{length} \) function. \text{fromIntegral} is a useful function that allows us to deal with this problem.

Folding a structure in reverse

Any fold can be run in the opposite direction with the help of the Dual monoid, which flips an existing monoid so that aggregation goes backwards.

```
newtype Dual a = Dual { getDual :: a }

instance Monoid m => Monoid (Dual m) where
```

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When the underlying monoid of a `foldMap` call is flipped with `Dual`, the fold runs backwards; the following `Reverse` type is defined in `Data.Functor.Reverse`:

```haskell
newtype Reverse t a = Reverse { getReverse :: t a }

instance Foldable t => Foldable (Reverse t) where
  foldMap f = getDual . foldMap (Dual . f) . getReverse
```

We can use this machinery to write a terse `reverse` for lists:

```haskell
reverse :: [a] -> [a]
reverse = toList . Reverse
```

An instance of Foldable for a binary tree

To instantiate `Foldable` you need to provide a definition for at least `foldMap` or `foldr`.

```haskell
data Tree a = Leaf
  | Node (Tree a) a (Tree a)

instance Foldable Tree where
  foldMap f Leaf = mempty
  foldMap f (Node l x r) = foldMap f l `mappend` f x `mappend` foldMap f r
  foldr f acc Leaf = acc
  foldr f acc (Node l x r) = foldr f (f x (foldr f acc r)) l
```

This implementation performs an in-order traversal of the tree.

```haskell
ghci> let myTree = Node (Node Leaf 'a' Leaf) 'b' (Node Leaf 'c' Leaf)
      --    +--'b'--+
      --    |       |
      -- +-'a'-+ +-'c'+
      -- |     | |     |
      -- *     * *     *

ghci> toList myTree
"abc"
```

The `DeriveFoldable` extension allows GHC to generate `Foldable` instances based on the structure of the type. We can vary the order of the machine-written traversal by adjusting the layout of the `Node` constructor.

```haskell
data Inorder a = ILeaf
  | INode (Inorder a) a (Inorder a) -- as before
deriving Foldable

data Preorder a = PrLeaf
  | PrNode a (Preorder a) (Preorder a)
```
deriving Foldable

data Postorder a = PoLeaf
    | PoNode (Postorder a) (Postorder a) a
    deriving Foldable

-- injections from the earlier Tree type
inorder :: Tree a -> Inorder a
inorder Leaf = ILeaf
inorder (Node l x r) = INode (inorder l) x (inorder r)

preorder :: Tree a -> Preorder a
preorder Leaf = PrLeaf
preorder (Node l x r) = PrNode x (preorder l) (preorder r)

postorder :: Tree a -> Postorder a
postorder Leaf = PoLeaf
postorder (Node l x r) = PoNode (postorder l) (postorder r) x

ghci> toList (inorder myTree)
"abc"
ghci> toList (preorder myTree)
"bac"
ghci> toList (postorder myTree)
"acb"

Flattening a Foldable structure into a list

toList flattens a Foldable structure t a into a list of as.

ghci> toList [7, 2, 9]  -- t ~ []
[7, 2, 9]
ghci> toList (Right 'a')  -- t ~ Either e
"a"
ghci> toList (Left "foo")  -- t ~ Either String
[]
ghci> toList (3, True)  -- t ~ (,) Int
[True]

toList is defined as being equivalent to:

class Foldable t where
    -- ...
    toList :: t a -> [a]
    toList = foldr (:) []

Performing a side-effect for each element of a Foldable structure

traverse_ executes an Applicative action for every element in a Foldable structure. It ignores the
action’s result, keeping only the side-effects. (For a version which doesn’t discard results, use
Traversable.)

-- using the Writer applicative functor (and the Sum monoid)
ghci> runWriter $ traverse_ (\x -> tell (Sum x)) [1,2,3]
((),Sum {getSum = 6})
-- using the IO applicative functor
ghci> traverse_ putStrLn (Right "traversing")
traversing
ghci> traverse_ putStrLn (Left False)
-- nothing printed

for_ is traverse_ with the arguments flipped. It resembles a foreach loop in an imperative language.

ghci> let greetings = ["Hello", "Bonjour", "Hola"]
ghci> :{
ghci|     for_ greetings $ \greeting -> do
ghci|         print (greeting ++ " Stack Overflow!")
ghci| :}
"Hello Stack Overflow!"
"Bonjour Stack Overflow!"
"Hola Stack Overflow!"

sequenceA_ collapses a Foldable full of Applicative actions into a single action, ignoring the result.

ghci> let actions = [putStrLn "one", putStrLn "two"]
ghci> sequenceA_ actions
one
two

traverse_ is defined as being equivalent to:

traverse_ :: (Foldable t, Applicative f) => (a -> f b) -> t a -> f ()
traverse_ f = foldr (\x action -> f x <*> action) (pure ())

sequenceA_ is defined as:

sequenceA_ :: (Foldable t, Applicative f) -> t (f a) -> f ()
sequenceA_ = traverse_ id

Moreover, when the Foldable is also a Functor, traverse_ and sequenceA_ have the following relationship:

traverse_ f = sequenceA_ . fmap f

Flattening a Foldable structure into a Monoid

foldMap maps each element of the Foldable structure to a Monoid, and then combines them into a single value.

foldMap and foldr can be defined in terms of one another, which means that instances of Foldable need only give a definition for one of them.

class Foldable t where
    foldMap :: Monoid m => (a -> m) -> t a -> m
    foldMap f = foldr (mappend . f) mempty
Example usage with the Product monoid:

```haskell
product :: (Num n, Foldable t) => t n -> n
product = getProduct . foldMap Product
```

**Definition of Foldable**

```haskell
class Foldable t where
  {-# MINIMAL foldMap | foldr #-}

  foldMap :: Monoid m => (a -> m) -> t a -> m
  foldMap f = foldr (mappend . f) mempty

  foldr :: (a -> b -> b) -> b -> t a -> b
  foldr f z t = appEndo (foldMap (Endo #. f) t) z

  -- and a number of optional methods
```

Intuitively (though not technically), Foldable structures are containers of elements which allow access to their elements in a well-defined order. The `foldMap` operation maps each element of the container to a `Monoid` and collapses them using the `Monoid` structure.

**Checking if a Foldable structure is empty**

`null` returns `True` if there are no elements in a foldable structure `t a`, and `False` if there is one or more. Structures for which `null` is `True` have a length of 0.

```haskell
ghci> null []
True
ghci> null [14, 29]
False
ghci> null Nothing
True
ghci> null (Right 'a')
False
ghci> null ('x', 3)
False
```

`null` is defined as being equivalent to:

```haskell
class Foldable t where
  -- ...
  null :: t a -> Bool
  null = foldr (\_ _ -> False) True
```

Read Foldable online: [https://riptutorial.com/haskell/topic/753/foldable](https://riptutorial.com/haskell/topic/753/foldable)
Chapter 22: Foreign Function Interface

Syntax

- foreign import ccall unsafe "foo" hFoo :: Int32 -> IO Int32 {- Imports a function named foo in some object file, and defines the symbol hFoo which can be called with Haskell code. -}

Remarks

While cabal has support for including a C and C++ libraries in a Haskell package, there are a few bugs. First, if you have data (rather than a function) defined in b.o that is used in a.o, and list the C-sources: a.c, b.c, then cabal will be unable to find the data. This is documented in #12152. A workaround when using cabal is to reorder the C-sources list to be C-sources: b.c, a.c. This may not work when using stack, because stack always links the C-sources alphabetically, regardless of the order in which you list them.

Another issue is that you must surround any C++ code in header (.h) files with #ifdef __cplusplus guards. This is because GHC doesn't understand C++ code in header files. You can still write C++ code in header files, but you must surround it with guards.

ccall refers to the calling convention; currently ccall and stdcall (Pascal convention) are supported. The unsafe keyword is optional; this reduces overhead for simple functions but may cause deadlocks if the foreign function blocks indefinitely or has insufficient permission to execute.

Examples

Calling C from Haskell

For performance reasons, or due to the existence of mature C libraries, you may want to call C code from a Haskell program. Here is a simple example of how you can pass data to a C library and get an answer back.

foo.c:

```c
#include <inttypes.h>

int32_t foo(int32_t a) {
    return a+1;
}
```

Foo.hs:

```haskell
import Data.Int

main :: IO ()
```
The `unsafe` keyword generates a more efficient call than 'safe', but requires that the C code never makes a callback to the Haskell system. Since `foo` is completely in C and will never call Haskell, we can use `unsafe`.

We also need to instruct cabal to compile and link in C source.

```haskell
foo.cabal:

name:                foo
version:             0.0.0.1
build-type:          Simple
extra-source-files:  *.c
cabal-version:       >= 1.10

executable foo
  default-language: Haskell2010
  main-is:       Foo.hs
  C-sources:     foc.c
  build-depends: base
```

Then you can run:

```
> cabal configure
> cabal build foo
> ./dist/build/foo/foo
42
```

**Passing Haskell functions as callbacks to C code.**

It is very common for C functions to accept pointers to other functions as arguments. Most popular example is setting an action to be executed when a button is clicked in some GUI toolkit library. It is possible to pass Haskell functions as C callbacks.

To call this C function:

```c
void event_callback_add (Object *obj, Object_Event_Cb func, const void *data)
```

we first import it to Haskell code:

```haskell
foreign import ccall "header.h event_callback_add"
  callbackAdd :: Ptr () -> FunPtr Callback -> Ptr () -> IO ()
```

Now looking at how `Object_Event_Cb` is defined in C header, define what `Callback` is in Haskell:

```haskell
type Callback = Ptr () -> Ptr () -> IO ()
```

Finally, create a special function that would wrap Haskell function of type `Callback` into a pointer
FunPtr Callback:

```haskell
foreign import ccall "wrapper"
    mkCallback :: Callback -> IO (FunPtr Callback)
```

Now we can register callback with C code:

```haskell
cbPtr <- mkCallback $ \objPtr dataPtr -> do
    -- callback code
    return ()
    callbackAdd cpPtr
```

It is important to free allocated FunPtr once you unregister the callback:

```haskell
freeHaskellFunPtr cbPtr
```

Read Foreign Function Interface online: https://riptutorial.com/haskell/topic/7256/foreign-function-interface
Chapter 23: Free Monads

Examples

Free monads split monadic computations into data structures and interpreters

For instance, a computation involving commands to read and write from the prompt:

First we describe the "commands" of our computation as a Functor data type

```haskell
{-# LANGUAGE DeriveFunctor #-}
data TeletypeF next = PrintLine String next | ReadLine (String -> next) deriving Functor
```

Then we use `Free` to create the "Free Monad over `TeletypeF" and build some basic operations.

```haskell
import Control.Monad.Free (Free, liftF, iterM)
type Teletype = Free TeletypeF
printLine :: String -> Teletype ()
printLine str = liftF (PrintLine str ())
readLine :: Teletype String
readLine = liftF (ReadLine id)
```

Since `Free f` is a Monad whenever `f` is a Functor, we can use the standard Monad combinators (including do notation) to build `Teletype` computations.

```haskell
import Control.Monad -- we can use the standard combinators
  echo :: Teletype ()
  echo = readLine >>= printLine
  mockingbird :: Teletype a
  mockingbird = forever echo

Finally, we write an "interpreter" turning `Teletype a` values into something we know how to work with like `IO a`

```haskell
interpretTeletype :: Teletype a -> IO a
interpretTeletype = foldFree run where
  run :: TeletypeF a -> IO a
  run (PrintLine str x) = putStrLn x >> return x
  run (ReadLine f) = fmap f getLine
```

https://riptutorial.com/
Which we can use to "run" the Teletype a computation in IO

```
> interpretTeletype mockingbird
hello
hello
goodbye
goodbye
this will go on forever
this will go on forever
```

Free Monads are like fixed points

Compare the definition of Free to that of Fix:

```
data Free f a = Return a
                | Free (f (Free f a))

newtype Fix f = Fix { unFix :: f (Fix f) }
```

In particular, compare the type of the Free constructor with the type of the Fix constructor. Free layers up a functor just like Fix, except that Free has an additional Return a case.

How do foldFree and iterM work?

There are some functions to help tear down Free computations by interpreting them into another monad m:

```
iterM :: (Functor f, Monad m) => (f (m a) -> m a) -> (Free f a -> m a)
foldFree :: Monad m => (forall x. f x -> m x) -> (Free f a -> m a)
```

What are they doing?

First let's see what it would take to tear down an interpret a Teletype a function into IO manually. We can see Free f a as being defined

```
data Free f a
    = Pure a
    | Free (f (Free f a))
```

The Pure case is easy:

```
interpretTeletype :: Teletype a -> IO a
interpretTeletype (Pure x) = return x
interpretTeletype (Free teletypeF) = _
```

Now, how to interpret a Teletype computation that was built with the Free constructor? We'd like to arrive at a value of type IO a by examining teletypeF :: TeletypeF (Teletype a). To start with, we'll write a function runIO :: TeletypeF a -> IO a which maps a single layer of the free monad to an IO action:

```
runIO :: TeletypeF a -> IO a
runIO (PrintLine msg x) = putStrLn msg *> return x
runIO (ReadLine k) = fmap k getLine
```
Now we can use `runIO` to fill in the rest of `interpretTeletype`. Recall that `teletypeF :: TeletypeF (Teletype a)` is a layer of the `TeletypeF` functor which contains the rest of the `Free` computation. We’ll use `runIO` to interpret the outermost layer (so we have `runIO teletypeF :: IO (Teletype a)`) and then use the `IO` monad’s `>>=` combinator to interpret the returned `Teletype a`.

```haskell
interpretTeletype :: Teletype a -> IO a
interpretTeletype (Pure x) = return x
interpretTeletype (Free teletypeF) = runIO teletypeF >>= interpretTeletype
```

The definition of `foldFree` is just that of `interpretTeletype`, except that the `runIO` function has been factored out. As a result, `foldFree` works independently of any particular base functor and of any target monad.

```haskell
foldFree :: Monad m => (forall x. f x -> m x) -> Free f a -> m a
foldFree eta (Pure x) = return x
foldFree eta (Free fa) = eta fa >>= foldFree eta
```

`foldFree` has a rank-2 type: `eta` is a natural transformation. We could have given `foldFree` a type of `Monad m => (f (Free f a) -> m (Free f a)) -> Free f a -> m a`, but that gives `eta` the liberty of inspecting the `Free` computation inside the `f` layer. Giving `foldFree` this more restrictive type ensures that `eta` can only process a single layer at a time.

`iterM` does give the folding function the ability to examine the subcomputation. The (monadic) result of the previous iteration is available to the next, inside `i`'s parameter. `iterM` is analogous to a `paramorphism` whereas `foldFree` is like a `catamorphism`.

```haskell
iterM :: (Monad m, Functor f) => (f (m a) -> m a) -> Free f a -> m a
iterM phi (Pure x) = return x
iterM phi (Free fa) = phi (fmap (iterM phi) fa)
```

The Freer monad

There’s an alternative formulation of the free monad called the Freer (or Prompt, or Operational) monad. The Freer monad doesn't require a Functor instance for its underlying instruction set, and it has a more recognisably list-like structure than the standard free monad.

The Freer monad represents programs as a sequence of atomic `instructions` belonging to the instruction set `i :: * -> *`. Each instruction uses its parameter to declare its return type. For example, the set of base instructions for the `State` monad are as follows:

```haskell
data StateI s a where
    Get :: StateI s s  -- the Get instruction returns a value of type 's'
    Put :: s -> StateI s ()  -- the Put instruction contains an 's' as an argument and returns ()
```

Sequencing these instructions takes place with the `:>>>` constructor. `:>>>` takes a single instruction returning an `a` and prepends it to the rest of the program, piping its return value into the continuation. In other words, given an instruction returning an `a`, and a function to turn an `a` into a program returning a `b`, `:>>>` will produce a program returning a `b`. 

https://riptutorial.com/
data Freer i a where
  Return :: a -> Freer i a
  (i >>=>) :: i a -> (a -> Freer i b) -> Freer i b

Note that a is existentially quantified in the \( \triangleright\triangleright= \) constructor. The only way for an interpreter to learn what a is is by pattern matching on the GADT i.

Aside: The co-Yoneda lemma tells us that Freer is isomorphic to Free. Recall the definition of the CoYoneda functor:

data CoYoneda i b where
  CoYoneda :: i a -> (a -> b) -> CoYoneda i b

Freer i is equivalent to Free (CoYoneda i). If you take Free’s constructors and set f ~ CoYoneda i, you get:

- Pure :: a -> Free (CoYoneda i) a
- Free :: CoYoneda i (Free (CoYoneda i) b) -> Free (CoYoneda i) b ~ i a -> (a -> Free (CoYoneda i) b) -> Free (CoYoneda i) b

from which we can recover Freer i’s constructors by just setting Freer i ~ Free (CoYoneda i).

Because CoYoneda i is a Functor for any i, Freer is a Monad for any i, even if i isn’t a Functor.

instance Monad (Freer i) where
  return = Return
  Return x >>= f = f x
  (i >>=> g >>= f = i >>=> fmap (>>= f) g -- using \`\`r\`\`'s instance of Functor, so fmap = (\.\))

Interpreters can be built for Freer by mapping instructions to some handler monad.

foldFreer :: Monad m => (forall x. i x -> m x) -> Freer i a -> m a
foldFreer eta (Return x) = return x
foldFreer eta (i >>=> f) = eta i >>= (foldFreer eta . f)

For example, we can interpret the Freer (StateI s) monad using the regular State s monad as a handler:

runFreerState :: Freer (StateI s) a -> s -> (a, s)
runFreerState = State.runState . foldFreer toState
  where toState :: StateI s a -> State s a
toState Get = State.get
toState (Put x) = State.put x

Read Free Monads online: https://riptutorial.com/haskell/topic/1290/free-monads
Chapter 24: Function call syntax

Introduction

Haskell's function call syntax, explained with comparisons to C-style languages where applicable. This is aimed at people who are coming to Haskell from a background in C-style languages.

Remarks

In general, the rule for converting a C-style function call to Haskell, in any context (assignment, return, or embedded in another call), is to replace the commas in the C-style argument list with whitespace, and move the opening parenthesis from the C-style call to contain the function name and its parameters.

If any expressions are wrapped entirely in parentheses, these (external) pairs of parentheses can be removed for readability, as they do not affect the meaning of the expression. There are some other circumstances where parentheses can be removed, but this only affects readability and maintainability.

Examples

Parentheses in a basic function call

For a C-style function call, e.g.

```c
plus(a, b); // Parentheses surrounding only the arguments, comma separated
```

Then the equivalent Haskell code will be

```haskell
(plus a b) -- Parentheses surrounding the function and the arguments, no commas
```

In Haskell, parentheses are not explicitly required for function application, and are only used to disambiguate expressions, like in mathematics; so in cases where the brackets surround all the text in the expression, the parentheses are actually not needed, and the following is also equivalent:

```haskell
plus a b -- no parentheses are needed here!
```

It is important to remember that while in C-style languages, the function

Parentheses in embedded function calls

In the previous example, we didn't end up needing the parentheses, because they did not affect the meaning of the statement. However, they are often necessary in more complex expression,
like the one below.

In C:

```c
plus(a, take(b, c));
```

In Haskell this becomes:

```haskell
(plus a (take b c))
-- or equivalently, omitting the outermost parentheses
plus a (take b c)
```

Note, that this is not equivalent to:

```haskell
plus a take b c -- Not what we want!
```

One might think that because the compiler knows that `take` is a function, it would be able to know that you want to apply it to the arguments `b` and `c`, and pass its result to `plus`. However, in Haskell, functions often take other functions as arguments, and little actual distinction is made between functions and other values; and so the compiler cannot assume your intention simply because `take` is a function.

And so, the last example is analogous to the following C function call:

```c
plus(a, take, b, c); // Not what we want!
```

**Partial application - Part 1**

In Haskell, functions can be partially applied; we can think of all functions as taking a single argument, and returning a modified function for which that argument is constant. To illustrate this, we can bracket functions as follows:

```haskell
(((plus) 1) 2)
```

Here, the function `plus` is applied to 1 yielding the function `((plus) 1)`, which is applied to 2, yielding the function `(((plus) 1) 2)`. Because `plus 1 2` is a function which takes no arguments, you can consider it a plain value; however in Haskell, there is little distinction between functions and values.

To go into more detail, the function `plus` is a function that adds its arguments. The function `plus 1` is a function that adds 1 to its argument. The function `plus 1 2` is a function that adds 1 to 2, which is always the value 3.

**Partial application - Part 2**

As another example, we have the function `map`, which takes a function and a list of values, and applies the function to each value of the list:
map :: (a -> b) -> [a] -> [b]

Let's say we want to increment each value in a list. You may decide to define your own function, which adds one to its argument, and map that function over your list

```haskell
addOne x = plus 1 x
map addOne [1,2,3]
```

but if you have another look at addOne’s definition, with parentheses added for emphasis:

```haskell
(addOne) x = ((plus) 1) x
```

The function addOne, when applied to any value x, is the same as the partially applied function plus 1 applied to x. This means the functions addOne and plus 1 are identical, and we can avoid defining a new function by just replacing addOne with plus 1, remembering to use parentheses to isolate plus 1 as a subexpression:

```haskell
map (plus 1) [1,2,3]
```

Read Function call syntax online: https://riptutorial.com/haskell/topic/9615/function-call-syntax
Chapter 25: Function composition

Remarks

Function composition operator \((.)\) is defined as

\[
(\cdot) : (b \to c) \to (a \to b) \to (a \to c)
\]

\[
(\cdot) \quad f \quad g \quad x = f (g x) \quad -- \text{or, equivalently,}\]

\[
(\cdot) \quad f \quad g = \lambda x \to f (g x)
\]

\[
(\cdot) = \lambda f \quad g \quad \to \lambda x \to f (g x)
\]

The type \((b \to c) \to (a \to b) \to (a \to c)\) can be written as \((b \to c) \to (a \to b) \to a \to c\) because the \(-\) in type signatures "associates" to the right, corresponding to the function application associating to the left,

\[
f \ g \ x \ y \ z \ \ldots \quad == \quad (((f \ g) \ x) \ y) \ z \ \ldots
\]

So the "dataflow" is from the right to the left: \(x\) "goes" into \(g\), whose result goes into \(f\), producing the final result:

\[
(\cdot) \quad f \quad g \quad x = r \quad \text{where} \quad r = f (g x)
\]

\[
-- g :: a \to b
\]

\[
-- f :: b \to c
\]

\[
-- x :: a
\]

\[
-- r :: c
\]

\[
(\cdot) \quad f \quad g = q \quad \text{where} \quad q = \lambda x \to f (g x)
\]

\[
-- g :: a \to b
\]

\[
-- f :: b \to c
\]

\[
-- q :: a \to c
\]

....

Syntactically, the following are all the same:

\[
(\cdot) \quad f \ g \ x = (f \ . \ g) \ x = (f \ .) \ g \ x = (\cdot \ g) \ f \ x
\]

which is easy to grasp as the "three rules of operator sections", where the "missing argument" just goes into the empty slot near the operator:

\[
(\cdot) \quad f \ g = (f \ . \ g) = (f \ .) \ g = (\cdot \ g) \ f
\]

\[
-- \ 1 \ 2 \ 3
\]

The \(x\), being present on both sides of the equation, can be omitted. This is known as eta-
contraction. Thus, the simple way to write down the definition for function composition is just

\[(f \cdot g) x = f (g x)\]

This of course refers to the "argument" \(x\); whenever we write just \((f \cdot g)\) without the \(x\) it is known as point-free style.

**Examples**

**Right-to-left composition**

\((.\) lets us compose two functions, feeding output of one as an input to the other:

\[(f \cdot g) x = f (g x)\]

For example, if we want to square the successor of an input number, we can write

\[\left((^2) \cdot \text{succ}\right) 1 \quad -- \quad 4\]

There is also \((<<<)\) which is an alias to \((.\). So,

\[\left(+ 1\right) <<< \text{sqrt} \, 25 \quad -- \quad 6\]

**Left-to-right composition**

`Control.Category` defines \((>>>)\), which, when specialized to functions, is

\[
\begin{align*}
\text{-- (>>>) :: Category cat => cat a b -> cat b c -> cat a c} \\
\text{-- (>>>) :: (->) a b -> (->) b c -> (->) a c} \\
\text{-- (>>>) :: (a -> b) -> (b -> c) -> (a -> c)} \\
(f \, >>\, g) \, x = g \, (f \, x)
\end{align*}
\]

Example:

```
sqrt >>> (+ 1) $ 25 \quad -- \quad 6.0
```

**Composition with binary function**

The regular composition works for unary functions. In the case of binary, we can define

\[
\begin{align*}
(f \, :\, g) \, x \, y = f \, (g \, x \, y) \quad & \text{-- which is also} \\
& = f \, ((g \, x) \, y) \\
& = (f \, \cdot \, g) \, x \, y \quad & \text{-- by definition of (.)} \\
& = (f \, \cdot \, ) \, (g \, x) \, y \\
& = ((f \, \cdot \, ) \, . \, g) \, x \, y
\end{align*}
\]

Thus, \((f \, :\, g) = ((f \, \cdot \, ) \, . \, g)\) by eta-contraction, and furthermore,
(.:) f g = ((f .) . g)
      = (.)(f .) g
      = (.)((.) f) g
      = (((.) .)(.)) f g

so (.:) = ((.) . (.), a semi-famous definition.

Examples:

(map (+1) .: filter) even [1..5]      --  [3,5]
(length   .: filter) even [1..5]      --  2

Read Function composition online: https://riptutorial.com/haskell/topic/4430/function-composition
Chapter 26: Functor

Introduction

Functor is the class of types \( f :: * \to * \) which can be covariantly mapped over. Mapping a function over a data structure applies the function to all the elements of the structure without changing the structure itself.

Remarks

A Functor can be thought of as a container for some value, or a computation context. Examples are `Maybe a` or `[a]`. The Typeclassopedia article has a good write-up of the concepts behind Functors.

To be considered a real Functor, an instance has to respect the 2 following laws:

Identity

\[
\text{fmap id} = \text{id}
\]

Composition

\[
\text{fmap (f \circ g)} = (\text{fmap f}) \circ (\text{fmap g})
\]

Examples

Common instances of Functor

Maybe

Maybe is a Functor containing a possibly-absent value:

```haskell
instance Functor Maybe where
  fmap f Nothing = Nothing
  fmap f (Just x) = Just (f x)
```

Maybe's instance of Functor applies a function to a value wrapped in a `Just`. If the computation has previously failed (so the `Maybe` value is a `Nothing`), then there's no value to apply the function to, so `fmap` is a no-op.

```
> fmap (+ 3) (Just 3)
Just 6
> fmap length (Just "mousetrap")

```

https://riptutorial.com/
We can check the functor laws for this instance using equational reasoning. For the identity law,

\[
\text{fmap id Nothing = Nothing} \quad \text{-- definition of fmap} \\
\text{id Nothing = Nothing} \quad \text{-- definition of id} \\
\text{fmap id (Just x) = Just x} \quad \text{-- definition of id} \\
\text{id (Just x) = Just x} \quad \text{-- definition of id}
\]

For the composition law,

\[
\text{(fmap f . fmap g) Nothing = Nothing} \quad \text{-- definition of (.)} \\
\text{fmap f (fmap g Nothing) = fmap g Nothing} \quad \text{-- definition of fmap} \\
\text{fmap (f . g) Nothing = fmap f Nothing} \quad \text{because Nothing = fmap f Nothing, for all f} \\
\text{(fmap f . fmap g) (Just x) = Just (f x)} \quad \text{-- definition of fmap} \\
\text{fmap f (fmap g (Just x)) = Just (f x)} \quad \text{-- definition of fmap} \\
\text{fmap f (Just (g x)) = Just (f (g x))} \quad \text{-- definition of fmap} \\
\text{(fmap f . fmap g) (Just x) = Just ((f . g) x) = fmap (f . g) (Just x)} \quad \text{-- definition of fmap}
\]

### Lists

Lists' instance of `Functor` applies the function to every value in the list in place.

```haskell
instance Functor [] where
  fmap f [] = []
  fmap f (x:xs) = f x : fmap f xs
```

This could alternatively be written as a list comprehension: \( \text{fmap f xs = [f x | x <- xs]} \).

This example shows that `fmap` generalises `map`. `map` only operates on lists, whereas `fmap` works on an arbitrary `Functor`.

The identity law can be shown to hold by induction:

```
  -- base case
  fmap id [] = [] \quad \text{-- definition of fmap} \\
  id [] = [] \quad \text{-- definition of id}
```

```
  -- inductive step
  fmap id (x:xs) = id x : fmap id xs \quad \text{-- definition of fmap}
```
and similarly, the composition law:

```
-- base case
(fmap f . fmap g) []
fmap f (fmap g []) -- definition of (.)
fmap f [] -- definition of fmap
[] -- definition of fmap
fmap (f . g) [] -- because [] = fmap f [], for all f

-- inductive step
(fmap f . fmap g) (x:xs)
fmap f (fmap g (x:xs)) -- definition of (.)
fmap f (g x : fmap g xs) -- definition of fmap
f (g x) : fmap f (fmap g xs) -- definition of fmap
(f . g) x : fmap f (fmap g xs) -- definition of (.)
(f . g) x : fmap (f . g) xs -- by the inductive hypothesis
fmap (f . g) xs -- definition of fmap
```

## Functions

Not every Functor looks like a container. Functions' instance of Functor applies a function to the return value of another function.

```
instance Functor ((->) r) where
  fmap f g = \x -> f (g x)
```

Note that this definition is equivalent to fmap = (.). So fmap generalises function composition.

Once more checking the identity law:

```
fmap id g
\x -> id (g x) -- definition of fmap
\x -> g x -- definition of id
g -- eta-reduction
id g -- definition of id
```

and the composition law:

```
(fmap f . fmap g) h
fmap f (fmap g h) -- definition of (.)
fmap f (\x -> g (h x)) -- definition of fmap
\y -> f ((\x -> g (h x)) y) -- definition of fmap
\y -> f (g (h y)) -- beta-reduction
\y -> (f . g) (h y) -- definition of (.)
fmap (f . g) h -- definition of fmap
```
class Functor f where
    fmap :: (a -> b) -> f a -> f b

One way of looking at it is that `fmap` lifts a function of values into a function of values in a context `f`.

A correct instance of `Functor` should satisfy the functor laws, though these are not enforced by the compiler:

    fmap id = id              -- identity
    fmap f . fmap g = fmap (f . g) -- composition

There's a commonly-used infix alias for `fmap` called `<$>`.

    infixl 4 <$>  
    ( <$> ) :: Functor f => (a -> b) -> f a -> f b  
    ( <$> ) = fmap

Replacing all elements of a Functor with a single value

The `Data.Functor` module contains two combinators, `<$>` and `>$`, which ignore all of the values contained in a functor, replacing them all with a single constant value.

    infixl 4 <$, $>  
    <$ :: Functor f => a -> f b -> f a  
    <$ = fmap . const  
    $> :: Functor f => f a -> b -> f b  
    $> = flip (<$)

`void` ignores the return value of a computation.

    void :: Functor f => f a -> f ()  
    void = (()) <$

Polynomial functors

There's a useful set of type combinators for building big `Functor`s out of smaller ones. These are instructive as example instances of `Functor`, and they're also useful as a technique for generic programming, because they can be used to represent a large class of common functors.

The identity functor

The identity functor simply wraps up its argument. It's a type-level implementation of the `I` combinator from SKI calculus.

    newtype I a = I a
instance Functor I where
  \( \text{fmap } f \ (I \ x) = I \ (f \ x) \)

I can be found, under the name of Identity, in the `Data.Functor.Identity` module.

---

**The constant functor**

The constant functor ignores its second argument, containing only a constant value. It's a type-level analogue of `const`, the \( K \) combinator from SKI calculus.

```haskell
newtype K c a = K c
```

Note that \( K \ c \ a \) doesn't contain any \( a \)-values; \( K \ () \) is isomorphic to `Proxy`. This means that \( K \)'s implementation of `fmap` doesn't do any mapping at all!

```haskell
instance Functor (K c) where
  \( \text{fmap } _\_ \ (K \ c) = K \ c \)
```

\( K \) is otherwise known as `Const`, from `Data.Functor.Const`.

The remaining functors in this example combine smaller functors into bigger ones.

---

**Functor products**

The functor product takes a pair of functors and packs them up. It's analogous to a tuple, except that while \( (,) \) :: \(*\) \rightarrow \(*\) \rightarrow \(*\) operates on types \(*\) \rightarrow \(*\) \rightarrow \(*\) \rightarrow \(*\)

```haskell
infixl 7 :*

data (f :*: g) a = f a :*: g a

instance (Functor f, Functor g) => Functor (f :*: g) where
  \( \text{fmap } f \ (fx :*: gy) = \text{fmap } f \ fx :*: \text{fmap } f \ gy \)
```

This type can be found, under the name `Product`, in the `Data.Functor.Product` module.

---

**Functor coproducts**

Just like \( :+\) is analogous to \( (,) \), \( :+\) is the functor-level analogue of `Either`.

```haskell
infixl 6 :+

data (f :+: g) a = InL (f a) | InR (g a)

instance (Functor f, Functor g) => Functor (f :+: g) where
  \( \text{fmap } f \ (\text{InL } fx) = \text{InL } (\text{fmap } f \ fx) \)
  \( \text{fmap } f \ (\text{InR } gy) = \text{InR } (\text{fmap } f \ gy) \)
```
Functor composition

Finally, :.:. works like a type-level (.), taking the output of one functor and plumbing it into the input of another.

```haskell
infixr 9 :.:
newtype (f :.:. g) a = Cmp (f (g a))

instance (Functor f, Functor g) => Functor (f :.:. g) where
  fmap f (Cmp fgx) = Cmp (fmap (fmap f) fgx)
```

The `Compose` type can be found in `Data.Functor.Compose`.

Polynomial functors for generic programming

`I, K, :+:,:+:` and `:.:` can be thought of as a kit of building blocks for a certain class of simple datatypes. The kit becomes especially powerful when you combine it with fixed points because datatypes built with these combinators are automatically instances of `Functor`. You use the kit to build a template type, marking recursive points using `I`, and then plug it into `Fix` to get a type that can be used with the standard zoo of recursion schemes.

<table>
<thead>
<tr>
<th>Name</th>
<th>As a datatype</th>
<th>Using the functor kit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pairs of values</td>
<td><code>data Pair a = Pair a a</code></td>
<td><code>type Pair = I :+: I</code></td>
</tr>
<tr>
<td>Two-by-two grids</td>
<td><code>type Grid a = Pair (Pair a)</code></td>
<td><code>type Grid = Pair :.:. Pair</code></td>
</tr>
<tr>
<td>Natural numbers</td>
<td>`data Nat = Zero</td>
<td>Succ Nat`</td>
</tr>
<tr>
<td>Lists</td>
<td>`data List a = Nil</td>
<td>Cons a (List a)`</td>
</tr>
<tr>
<td>Binary trees</td>
<td>`data Tree a = Leaf</td>
<td>Node (Tree a) a (Tree a)`</td>
</tr>
<tr>
<td>Rose trees</td>
<td><code>data Rose a = Rose a (List (Rose a))</code></td>
<td><code>type Rose a = Fix (K a :+: List :.:. I)</code></td>
</tr>
</tbody>
</table>

This "kit" approach to designing datatypes is the idea behind *generic programming* libraries such as `generics-sop`. The idea is to write generic operations using a kit like the one presented above, and then use a type class to convert arbitrary datatypes to and from their generic representation:

```haskell
class Generic a where
  type Rep a -- a generic representation built using a kit
```
Functors in Category Theory

A Functor is defined in category theory as a structure-preserving map (a ‘homomorphism’) between categories. Specifically, (all) objects are mapped to objects, and (all) arrows are mapped to arrows, such that the category laws are preserved.

The category in which objects are Haskell types and morphisms are Haskell functions is called Hask. So a functor from Hask to Hask would consist of a mapping of types to types and a mapping from functions to functions.

The relationship that this category theoretic concept bears to the Haskell programming construct Functor is rather direct. The mapping from types to types takes the form of a type \( f :: \ast \rightarrow \ast \), and the mapping from functions to functions takes the form of a function \( \text{fmap} :: (a \rightarrow b) \rightarrow (f a \rightarrow f b) \). Putting those together in a class,

```haskell
class Functor (f :: \ast \rightarrow \ast) where
  fmap :: (a -> b) -> f a -> f b
```

fmap is an operation that takes a function (a type of morphism), :: a -> b, and maps it to another function, :: f a -> f b. It is assumed (but left to the programmer to ensure) that instances of Functor are indeed mathematical functors, preserving Hask’s categorical structure:

```haskell
fmap (id {- :: a -> a -}) == id {- :: f a -> f a -}
fmap (h . g) == fmap h . fmap g
```

fmap lifts a function :: a -> b into a subcategory of Hask in a way that preserves both the existence of any identity arrows, and the associativity of composition.

The Functor class only encodes endofunctors on Hask. But in mathematics, functors can map between arbitrary categories. A more faithful encoding of this concept would look like this:

```haskell
class Category c where
  id :: c i i
  (.) :: c j k -> c i j -> c i k

class (Category c1, Category c2) => CFunctor c1 c2 f where
  cfmap :: c1 a b -> c2 (f a) (f b)
```

The standard Functor class is a special case of this class in which the source and target categories are both Hask. For example,

```haskell
instance Category (->) where
  id = \x -> x
  f . g = \x -> f (g x)

instance CFunctor (->) (->) [] where
```

https://riptutorial.com/
Deriving Functor

The `DeriveFunctor` language extension allows GHC to generate instances of `Functor` automatically.

```haskell
{-# LANGUAGE DeriveFunctor #-}

data List a = Nil | Cons a (List a) deriving Functor

-- instance Functor List where       -- automatically defined
--   fmap f Nil = Nil
--   fmap f (Cons x xs) = Cons (f x) (fmap f xs)

map :: (a -> b) -> List a -> List b
map = fmap

Read Functor online: https://riptutorial.com/haskell/topic/3800/functor
```
Chapter 27: Generalized Algebraic Data Types

Examples

Basic Usage

When the GADTs extension is enabled, besides regular data declarations, you can also declare generalized algebraic datatypes as follows:

```haskell
data DataType a where
  Constr1 :: Int -> a -> Foo a -> DataType a
  Constr2 :: Show a => a -> DataType a
  Constr3 :: DataType Int
```

A GADT declaration lists the types of all constructors a datatype has, explicitly. Unlike regular datatype declarations, the type of a constructor can be any N-ary (including nullary) function that ultimately results in the datatype applied to some arguments.

In this case we've declared that the type `DataType` has three constructors: `Constr1`, `Constr2` and `Constr3`.

The `Constr1` constructor is no different from one declared using a regular data declaration:

```haskell
data DataType a = Constr1 Int a (Foo a) | ...
```

`Constr2` however requires that `a` has an instance of `Show`, and so when using the constructor the instance would need to exist. On the other hand, when pattern-matching on it, the fact that `a` is an instance of `Show` comes into scope, so you can write:

```haskell
foo :: DataType a -> String
foo val = case val of
  Constr2 x -> show x
  ...
```

Note that the `Show a` constraint doesn't appear in the type of the function, and is only visible in the code to the right of `->`.

`Constr3` has type `DataType Int`, which means that whenever a value of type `DataType a` is a `Constr3`, it is known that `a ~ Int`. This information, too, can be recovered with a pattern match.

Read Generalized Algebraic Data Types online:
Chapter 28: GHCJS

Introduction

GHCJS is a Haskell to JavaScript compiler that uses the GHC API.

Examples

Running "Hello World!" with Node.js

ghcjs can be invoked with the same command line arguments as ghc. The generated programs can be run directly from the shell with Node.js and SpiderMonkey jsshell. for example:

```
$ ghcjs -o helloWorld helloWorld.hs
$ node helloWorld.js.exe/all.js
Hello world!
```

Read GHCJS online: https://riptutorial.com/haskell/topic/9260/ghcjs
Chapter 29: Google Protocol Buffers

Remarks

To use Protocol Buffers with Haskell you should install the htprotoc package:

1. Clone the project from Github
2. Use Stack to build and install

You should now find the hprotoc executable in $HOME/.local/bin/.

Examples

Creating, building and using a simple .proto file

Let us first create a simple .proto file person.proto

```haskell
package Protocol;

message Person {
  required string firstName = 1;
  required string lastName = 2;
  optional int32 age = 3;
}
```

After saving we can now create the Haskell files which we can use in our project by running

```
$HOME/.local/bin/hprotoc --proto_path=. --haskell_out=. person.proto
```

We should get an output similar to this:

```
Loading filepath: "<path-to-project>/person.proto"
All proto files loaded
Haskell name mangling done
Recursive modules resolved
./Protocol/Person.hs
./Protocol.hs
Processing complete, have a nice day.
```

hprotoc will create a new folder Protocol in the current directory with Person.hs which we can simply import into our haskell project:

```haskell
import Protocol (Person)
```

As a next step, if using Stack add

```
protocol-buffers
, protocol-buffers-descriptor
```
to build-depends: and

Protocol

to exposed-modules in your .cabal file.

If we get now a incoming message from a stream, the message will have the type \texttt{ByteString}.

In order to transform the \texttt{ByteString} (which obviously should contain encoded "Person" data) into our Haskell data type, we need to call the function \texttt{messageGet} which we import by

\begin{verbatim}
import Text.ProtocolBuffers (messageGet)
\end{verbatim}

which enables to create a value of type \texttt{Person} using:

\begin{verbatim}
transformRawPerson :: ByteString -> Maybe Person
transformRawPerson raw = case messageGet raw of
  Left   _           -> Nothing
  Right (person, _)  -> Just person
\end{verbatim}

Read Google Protocol Buffers online: https://riptutorial.com/haskell/topic/5018/google-protocol-buffers
Chapter 30: Graphics with Gloss

Examples

Installing Gloss

Gloss is easily installed using the Cabal tool. Having installed Cabal, one can run `cabal install gloss` to install Gloss.

Alternatively the package can be built from source, by downloading the source from Hackage or GitHub, and doing the following:

1. Enter the `gloss/gloss-rendering/` directory and do `cabal install`
2. Enter the `gloss/gloss/` directory and once more do `cabal install`

Getting something on the screen

In Gloss, one can use the `display` function to create very simple static graphics.

To use this one needs to first `import Graphics.Gloss`. Then in the code there should the following:

```haskell
main :: IO ()
main = display window background drawing
```

`window` is of type `Display` which can be constructed in two ways:

```haskell
-- Defines window as an actual window with a given name and size
window = InWindow name (width, height) (0,0)

-- Defines window as a fullscreen window
window = FullScreen
```

Here the last argument `(0,0)` in `InWindow` marks the location of the top left corner.

**For versions older than 1.11:** In older versions of Gloss `FullScreen` takes another argument which is meant to be the size of the frame that gets drawn on which in turn gets stretched to fullscreen-size, for example: `FullScreen (1024,768)`

`background` is of type `Color`. It defines the background color, so it's as simple as:

```haskell
background = white
```

Then we get to the drawing itself. Drawings can be very complex. How to specify these will be covered elsewhere ([one can refer to this for the moment][1]), but it can be as simple as the following circle with a radius of 80:

```haskell
drawing = Circle 80
```
Summarizing example

As more or less stated in the documentation on Hackage, getting something on the screen is as easy as:

```
import Graphics.Gloss

main :: IO ()
main = display window background drawing
  where
      window = InWindow "Nice Window" (200, 200) (0, 0)
      background = white
      drawing = Circle 80
```

Read Graphics with Gloss online: https://riptutorial.com/haskell/topic/5570/graphics-with-gloss
Chapter 31: Gtk3

Syntax

- obj <- < widgetName >New -- How widgets (e.g. Windows, Buttons, Grids) are created
- set < widget > [ < attributes > ] -- Set attributes as defined as Attr self in widget documentation (e.g. buttonLabel)
- on < widget > < event > < IO action > -- Adding an IO action to a widgets Signal self (e.g. buttonActivated)

Remarks

On many Linux distributions, the Haskell Gtk3 library is available as a package in the systems package manager (e.g. libghc-gtk in Ubuntu’s APT). However, for some developers it might be preferable to use a tool like stack to manage isolated environments, and have Gtk3 installed via cabal instead of via an global installation by the systems package manager. For this option, gtk2hs-buildtools is required. Run cabal install gtk2hs-buildtools before adding gtk, gtk3 or any other Gtk-based Haskell libraries to your projects build-depends entry in your cabal file.

Examples

Hello World in Gtk

This example show how one may create a simple "Hello World" in Gtk3, setting up a window and button widgets. The sample code will also demonstrate how to set different attributes and actions on the widgets.

```haskell
module Main (Main.main) where

import Graphics.UI.Gtk

main :: IO ()
main = do
  initGUI
  window <- windowNew
  on window objectDestroy mainQuit
  set window [ containerBorderWidth := 10, windowTitle := "Hello World" ]
  button <- buttonNew
  set button [ buttonLabel := "Hello World" ]
  on button buttonActivated $ do
    putStrLn "A \"clicked\"-handler to say \"destroy\"
    widgetDestroy window
  set window [ containerChild := button ]
  widgetShowAll window
  mainGUI -- main loop
```

https://riptutorial.com/
Read Gtk3 online: https://riptutorial.com/haskell/topic/7342/gtk3
Chapter 32: Higher-order functions

Remarks

Higher Order Functions are functions that take functions as parameters and/or return functions as their return values.

Examples

Basics of Higher Order Functions

Review Partial Application before proceeding.

In Haskell, a function that can take other functions as arguments or return functions is called a higher-order function.

The following are all higher-order functions:

```
map       :: (a -> b) -> [a] -> [b]
filter    :: (a -> Bool) -> [a] -> [a]
takeWhile :: (a -> Bool) -> [a] -> [a]
dropWhile :: (a -> Bool) -> [a] -> [a]
iterate   :: (a -> a) -> a -> [a]
zipWith   :: (a -> b -> c) -> [a] -> [b] -> [c]
scanr     :: (a -> b -> b) -> b -> [a] -> [b]
scanl     :: (b -> a -> b) -> b -> [a] -> [b]
```

These are particularly useful in that they allow us to create new functions on top of the ones we already have, by passing functions as arguments to other functions. Hence the name, higher-order functions.

Consider:

```
Prelude> :t (map (+3))
(map (+3)) :: Num b => [b] -> [b]
Prelude> :t (map (== 'c'))
(map (== 'c')) :: [Char] -> [Bool]
Prelude> :t (map zipWith)
(map zipWith) :: ([a -> b -> c] -> [[a] -> [b] -> [c]])
```

This ability to easily create functions (like e.g. by partial application as used here) is one of the features that makes functional programming particularly powerful and allows us to derive short, elegant solutions that would otherwise take dozens of lines in other languages. For example, the following function gives us the number of aligned elements in two lists.

```
aligned :: [a] -> [a] -> Int
```
Lambda Expressions

Lambda expressions are similar to anonymous functions in other languages.

Lambda expressions are open formulas which also specify variables which are to be bound. Evaluation (finding the value of a function call) is then achieved by substituting the bound variables in the lambda expression’s body, with the user supplied arguments. Put simply, lambda expressions allow us to express functions by way of variable binding and substitution.

Lambda expressions look like

\x \rightarrow \text{let } \{y = \ldots x \ldots\} \text{ in } y

Within a lambda expression, the variables on the left-hand side of the arrow are considered bound in the right-hand side, i.e. the function's body.

Consider the mathematical function

\[ f(x) = x^2 \]

As a Haskell definition it is

\[
\begin{align*}
  f & \quad x = x^2 \\
  f & \quad \text{\( \rightarrow \) } x^2
\end{align*}
\]

which means that the function \( f \) is equivalent to the lambda expression \( \lambda x \rightarrow x^2 \).

Consider the parameter of the higher-order function \( \text{map} \), that is a function of type \( \text{a} \rightarrow \text{b} \). In case it is used only once in a call to \( \text{map} \) and nowhere else in the program, it is convenient to specify it as a lambda expression instead of naming such a throwaway function. Written as a lambda expression,

\[
\lambda x \rightarrow \text{let } \{y = \ldots x \ldots\} \text{ in } y
\]

\( x \) holds a value of type \( \text{a} \), \( \ldots x \ldots \) is a Haskell expression that refers to the variable \( x \), and \( y \) holds a value of type \( \text{b} \). So, for example, we could write the following

map (\x \rightarrow x + 3)
map (\(\langle x, y \rangle \rightarrow x \ast y\))
map (\xs \rightarrow 'c':xs) ["apples", "oranges", "mangos"]
map (\f \rightarrow \text{\( \text{zipWith f [1..5] [1..5]}\)} [(+), (*)], (-])

Currying

https://riptutorial.com/
In Haskell, all functions are considered curried: that is, all functions in Haskell take just one argument.

Let's take the function `div`:

```haskell
div :: Int -> Int -> Int
```

If we call this function with 6 and 2 we unsurprisingly get 3:

```haskell
Prelude> div 6 2
3
```

However, this doesn't quite behave in the way we might think. First `div 6` is evaluated and returns a function of type `Int -> Int`. This resulting function is then applied to the value 2 which yields 3.

When we look at the type signature of a function, we can shift our thinking from "takes two arguments of type `Int" to "takes one `Int and returns a function that takes an `Int". This is reaffirmed if we consider that arrows in the type notation associate to the right, so `div` can in fact be read thus:

```haskell
div :: Int -> (Int -> Int)
```

In general, most programmers can ignore this behaviour at least while they're learning the language. From a theoretical point of view, "formal proofs are easier when all functions are treated uniformly (one argument in, one result out)."

Read Higher-order functions online: https://riptutorial.com/haskell/topic/4539/higher-order-functions
Chapter 33: Infix operators

Remarks

Most Haskell functions are called with the function name followed by arguments (prefix notation). For functions that accept two arguments like (+), it sometimes makes sense to provide an argument before and after the function (infix).

Examples

Prelude

Logical

&& is logical AND, || is logical OR.

== is equality, /= non-equality, < / <= lesser and > / >= greater operators.

Arithmetic operators

The numerical operators +, - and / behave largely as you’d expect. (Division works only on fractional numbers to avoid rounding issues – integer division must be done with quot or div). More unusual are Haskell's three exponentiation operators:

• ^ takes a base of any number type to a non-negative, integral power. This works simply by (fast) iterated multiplication. E.g.

4^5 = (4*4)*(4*4)*4

• ^^ does the same in the positive case, but also works for negative exponents. E.g.

3^^(-2) = 1 / (2*2)

Unlike ^, this requires a fractional base type (i.e. 4^^5 :: Int will not work, only 4^5 :: Int or 4^^5 :: Rational).

• ** implements real-number exponentiation. This works for very general arguments, but is more computationally expensive than ^ or ^^, and generally incurs small floating-point errors.

2**pi = exp (pi * log 2)
Lists

There are two concatenation operators:

- `:` (pronounced `cons`) prepends a single argument before a list. This operator is actually a constructor and can thus also be used to pattern match (“inverse construct”) a list.

- `++` concatenates entire lists.

```
[1,2] ++ [3,4] = 1 : 2 : [3,4] = 1 : [2,3,4] = [1,2,3,4]
```

`!!` is an indexing operator.

```
[0, 10, 20, 30, 40] !! 3 = 30
```

Note that indexing lists is inefficient (complexity $O(n)$ instead of $O(1)$ for arrays or $O(\log n)$ for maps); it’s generally preferred in Haskell to deconstruct lists by folding or pattern matching instead of indexing.

Control flow

- `$` is a function application operator.

```
f $ x = f x
    = f(x)  -- disapproved style
```

This operator is mostly used to avoid parentheses. It also has a strict version `$!$, which forces the argument to be evaluated before applying the function.

- `.` composes functions.

```
(f . g) x = f (g x) = f $ g x
```

- `>>` sequences monadic actions. E.g. `writeFile "foo.txt" "bla" >> putStrLn "Done."` will first write to a file, then print a message to the screen.

- `>>=` does the same, while also accepting an argument to be passed from the first action to the following. `readLn >>= \x -> print (x^2)` will wait for the user to input a number, then output the square of that number to the screen.

Custom operators

In Haskell, you can define any infix operator you like. For example, I could define the list-enveloping operator as

```
https://riptutorial.com/ 113
```
You should always give such operators a **fixity declaration**, like

```haskell
infixr 5 >++
```

(which would mean `>++` binds as tightly as `++` and `:`).

### Finding information about infix operators

Because infixes are so common in Haskell, you will regularly need to look up their signature etc.. Fortunately, this is just as easy as for any other function:

- The Haskell search engines `Hayoo` and `Hoogle` can be used for infix operators, like for anything else that's defined in some library.
- In GHCi or IHaskell, you can use the `:i` and `:t` (info and type) directives to learn the basic properties of an operator. For example,

```haskell
Prelude> :i +
class Num a where
  (+) :: a -> a -> a
  ...
  -- Defined in 'GHC.Num'
infixl 6 +

Prelude> :i ^^
(^^^) :: (Fractional a, Integral b) -> a -> b -> a
  -- Defined in 'GHC.Real'
infixr 8 ^
```

This tells me that `^^` binds more tightly than `+`, both take numerical types as their elements, but `^^` requires the exponent to be integral and the base to be fractional. The less verbose `:t` requires the operator in parentheses, like

```haskell
Prelude> :t (==)
(==) :: Eq a -> a -> a -> Bool
```

Read Infix operators online: https://riptutorial.com/haskell/topic/6792/infix-operators
Chapter 34: IO

Examples

Reading all contents of standard input into a string

```haskell
main = do
    input <- getContents
    putStrLn input
```

Input:

This is an example sentence.
And this one is, too!

Output:

This is an example sentence.
And this one is, too!

Note: This program will actually print parts of the output before all of the input has been fully read in. This means that, if, for example, you use `getContents` over a 50MiB file, Haskell's lazy evaluation and garbage collector will ensure that only the parts of the file that are currently needed (read: indispensable for further execution) will be loaded into memory. Thus, the 50MiB file won't be loaded into memory at once.

Reading a line from standard input

```haskell
main = do
    line <- getLine
    putStrLn line
```

Input:

This is an example.

Output:

This is an example.

Parsing and constructing an object from standard input

```haskell
readFloat :: IO Float
readFloat =
    fmap read getLine
```
main :: IO ()
main = do
  putStrLn "Type the first number: 
  first <- readFloat

  putStrLn "Type the second number: 
  second <- readFloat

  putStrLn $ show first ++ " + " ++ show second ++ " = " ++ show ( first + second )

Input:
Type the first number: 9.5
Type the second number: -2.02

Output:
9.5 + -2.02 = 7.48

Reading from file handles

Like in several other parts of the I/O library, functions that implicitly use a standard stream have a counterpart in System.IO that performs the same job, but with an extra parameter at the left, of type Handle, that represents the stream being handled. For instance, getLine :: IO String has a counterpart hGetLine :: Handle -> IO String.

import System.IO( Handle, FilePath, IOMode( ReadMode ),
                  openFile, hGetLine, hPutStr, hClose, hIsEOF, stderr )

import Control.Monad( when )

dumpFile :: Handle -> FilePath -> Integer -> IO ()
dumpFile handle filename lineNumber = do
  -- show file contents line by line
  end <- hIsEOF handle
  when ( not end ) $ do
    line <- hGetLine handle
    putStrLn $ filename ++ ":" ++ show lineNumber ++ ": " ++ line
    dumpFile handle filename $ lineNumber + 1

main :: IO ()
main = do
  hPutStr stderr "Type a filename: 
  filename <- getLine
  handle <- openFile filename ReadMode
  dumpFile handle filename 1
  hClose handle

Contents of file example.txt:
This is an example.
Hello, world!
This is another example.

Input:

Type a filename: example.txt

Output:

example.txt:1: This is an example.
example.txt:2: Hello, world!
example.txt:3: This is another example

Checking for end-of-file conditions

A bit counter-intuitive to the way most other languages' standard I/O libraries do it, Haskell's \texttt{isEOF} does not require you to perform a read operation before checking for an EOF condition; the runtime will do it for you.

\begin{verbatim}
import System.IO(isEOF)

eofTest :: Int -> IO ()
eofTest line = do
  end <- isEOF
  if end then
    putStrLn $ "End-of-file reached at line \" ++ show line ++ \\".
  else do
    getline
    eofTest $ line + 1

main :: IO ()
main =
  eofTest 1
\end{verbatim}

Input:

Line #1.
Line #2.
Line #3.

Output:

End-of-file reached at line 4.

Reading words from an entire file

In Haskell, it often makes sense \textit{not to bother with file handles} at all, but simply read or write an entire file straight from disk to memory, and do all the partitioning/processing of the text with the
pure string data structure. This avoids mixing IO and program logic, which can greatly help avoiding bugs.

```haskell
-- | The interesting part of the program, which actually processes data
-- but doesn't do any IO!
reverseWords :: String -> [String]
reverseWords = reverse . words

-- | A simple wrapper that only fetches the data from disk, lets
-- 'reverseWords' do its job, and puts the result to stdout.
main :: IO ()
main = do
    content <- readFile "loremipsum.txt"
    mapM_ putStrLn $ reverseWords content
```

If `loremipsum.txt` contains

```plaintext
Lorem ipsum dolor sit amet,
consectetur adipiscing elit
```

then the program will output

```plaintext
elit
adipiscing
consectetur
amet,
sit
dolor
ipsum
Lorem
```

Here, `mapM_` went through the list of all words in the file, and printed each of them to a separate line with `putStrLn`.

†If you think this is wasteful on memory, you have a point. Actually, Haskell's laziness can often avoid that the entire file needs to reside in memory simultaneously... but beware, this kind of lazy IO causes its own set of problems. For performance-critical applications, it often makes sense to enforce the entire file to be read at once, strictly; you can do this with the `Data.Text` version of `readFile`.

**IO defines your program's `main` action**

To make a Haskell program executable you must provide a file with a `main` function of type `IO ()`

```haskell
main :: IO ()
main = putStrLn "Hello world!"
```

When Haskell is compiled it examines the IO data here and turns it into a executable. When we run this program it will print `Hello world!`.

If you have values of type `IO a` other than `main` they won't do anything.
Compiling this program and running it will have the same effect as the last example. The code in `other` is ignored.

In order to make the code in `other` have runtime effects you have to compose it into `main`. No `IO` values not eventually composed into `main` will have any runtime effect. To compose two `IO` values sequentially you can use `do`-notation:

```
other :: IO ()
other = putStrLn "I will get printed... but only at the point where I'm composed into main"

main :: IO ()
main = do
  putStrLn "Hello world!"
  other
```

When you compile and run this program it outputs

```
Hello world!
I will get printed... but only at the point where I'm composed into main
```

Note that the order of operations is described by how `other` was composed into `main` and not the definition order.

Role and Purpose of IO

Haskell is a pure language, meaning that expressions cannot have side effects. A side effect is anything that the expression or function does other than produce a value, for example, modify a global counter or print to standard output.

In Haskell, side-effectful computations (specifically, those which can have an effect on the real world) are modelled using `IO`. Strictly speaking, `IO` is a type constructor, taking a type and producing a type. For example, `IO Int` is the type of an I/O computation producing an `Int` value. The `IO` type is abstract, and the interface provided for `IO` ensures that certain illegal values (that is, functions with non-sensical types) cannot exist, by ensuring that all built-in functions which perform I/O have a return type enclosed in `IO`.

When a Haskell program is run, the computation represented by the Haskell value named `main`, whose type can be `IO x` for any type `x`, is executed.

Manipulating IO values

There are many functions in the standard library providing typical `IO` actions that a general purpose programming language should perform, such as reading and writing to file handles.
General IO actions are created and combined primarily with two functions:

\[
(\gg\gg) :: \text{IO } a \rightarrow (a \rightarrow \text{IO } b) \rightarrow \text{IO } b
\]

This function (typically called \textit{bind}) takes an IO action and a function which returns an IO action, and produces the IO action which is the result of applying the function to the value produced by the first IO action.

\[
\text{return} :: a \rightarrow \text{IO } a
\]

This function takes any value (i.e., a pure value) and returns the IO computation which does no IO and produces the given value. In other words, it is a no-op I/O action.

There are additional general functions which are often used, but all can be written in terms of the two above. For example, \[
(\gg) :: \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } b
\] is similar to \(\gg\gg\) but the result of the first action is ignored.

A simple program greeting the user using these functions:

```haskell
main :: IO ()
main =
    putStrLn "What is your name?" >>
    getLine >>= \name ->
    putStrLn ("Hello " ++ name ++ "!")
```

This program also uses \texttt{putStrLn :: String \rightarrow IO ()} and \texttt{getLine :: IO String}.

Note: the types of certain functions above are actually more general than those types given (namely \(\gg\gg\), \(\gg\) and \texttt{return}).

### IO semantics

The IO type in Haskell has very similar semantics to that of imperative programming languages. For example, when one writes \(s_1 ; s_2\) in an imperative language to indicate executing statement \(s_1\), then statement \(s_2\), one can write \(s_1 \gg s_2\) to model the same thing in Haskell.

However, the semantics of IO diverge slightly of what would be expected coming from an imperative background. The \texttt{return} function \textit{does not} interrupt control flow - it has no effect on the program if another IO action is run in sequence. For example, \texttt{return () \gg putStrLn "boom"} correctly prints "boom" to standard output.

The formal semantics of IO can given in terms of simple equalities involving the functions in the previous section:

\[
\text{return } x \gg\gg f = f x, \forall f x
\]

\[
y \gg\gg \text{return } = \text{return } y, \forall y
\]

https://riptutorial.com/
These laws are typically referred to as left identity, right identity, and composition, respectively. They can be stated more naturally in terms of the function

\[
(f \gg g) x = (f x) \gg g
\]

as follows:

\[
\begin{align*}
\text{return} & \gg f = f, \forall f \\
f & \gg \text{return} = f, \forall f \\
(f \gg g) \gg h & = f \gg (g \gg h), \forall f, g, h
\end{align*}
\]

### Lazy IO

Functions performing I/O computations are typically strict, meaning that all preceding actions in a sequence of actions must be completed before the next action is begun. Typically this is useful and expected behaviour - `putStrLn "X" >> putStrLn "Y"` should print "XY". However, certain library functions perform I/O lazily, meaning that the I/O actions required to produce the value are only performed when the value is actually consumed. Examples of such functions are `getContents` and `readFile`. Lazy I/O can drastically reduce the performance of a Haskell program, so when using library functions, care should be taken to note which functions are lazy.

### IO and do notation

Haskell provides a simpler method of combining different IO values into larger IO values. This special syntax is known as do notation* and is simply syntactic sugar for usages of the `>>=`, `>>` and `return` functions.

The program in the previous section can be written in two different ways using do notation, the first being layout-sensitive and the second being layout insensitive:

```haskell
main = do
  putStrLn "What is your name?"
  name <- getLine
  putStrLn ("Hello " ++ name ++ ")"

main = do { putStrLn "What is your name?" ;
            name <- getLine ;
            putStrLn ("Hello " ++ name ++ ")"
          }
```

All three programs are exactly equivalent.
*Note that do notation is also applicable to a broader class of type constructors called monads.

**Getting the 'a' "out of" 'IO a'**

A common question is "I have a value of IO a, but I want to do something to that a value: how do I get access to it?" How can one operate on data that comes from the outside world (for example, incrementing a number typed by the user)?

The point is that if you use a pure function on data obtained impurely, then the result is still impure. It depends on what the user did! A value of type IO a stands for a "side-effecting computation resulting in a value of type a" which can only be run by (a) composing it into main and (b) compiling and executing your program. For that reason, there is no way within pure Haskell world to "get the a out".

Instead, we want to build a new computation, a new IO value, which makes use of the a value at runtime. This is another way of composing IO values and so again we can use do-notation:

```haskell
-- assuming
myComputation :: IO Int
getMessage :: Int -> String
getMessage int = "My computation resulted in: " ++ show int
newComputation :: IO ()
newComputation = do
  int <- myComputation       -- we "bind" the result of myComputation to a name, 'int'
  putStrLn $ getMessage int   -- 'int' holds a value of type Int
```

Here we're using a pure function (getMessage) to turn an Int into a String, but we're using do notation to make it be applied to the result of an IO computation myComputation when (after) that computation runs. The result is a bigger IO computation, newComputation. This technique of using pure functions in an impure context is called lifting.

**Writing to stdout**

Per the Haskell 2010 Language Specification the following are standard IO functions available in Prelude, so no imports are required to use them.

```haskell
putChar :: Char -> IO () - writes a char to stdout

Prelude> putChar 'a'
aPrelude> -- Note, no new line

putStr :: String -> IO () - writes a String to stdout

Prelude> putStr "This is a string!"
This is a string!Prelude> -- Note, no new line

putStrLn :: String -> IO () - writes a String to stdout and adds a new line
```

https://riptutorial.com/
Prelude> putStrLn "Hi there, this is another String!"
Hi there, this is another String!

print :: Show a => a -> IO () - writes a an instance of Show to stdout

Prelude> print "hi"
"hi"
Prelude> print 1
1
Prelude> print 'a'
'a'
Prelude> print (Just 'a')  -- Maybe is an instance of the `Show` type class
Just 'a'
Prelude> print Nothing
Nothing

Recall that you can instantiate Show for your own types using deriving:

-- In ex.hs
data Person = Person { name :: String } deriving Show

main = print (Person "Alex")  -- Person is an instance of `Show`, thanks to `deriving`

Loading & running in GHCi:

Prelude> :load ex.hs
[1 of 1] Compiling ex             ( ex.hs, interpreted )
Ok, modules loaded: ex.
*Main> main -- from ex.hs
Person {name = "Alex"}

Reading from `stdin`

As-per the Haskell 2010 Language Specification, the following are standard IO functions available in Prelude, so no imports are required to use them.

getChar :: IO Char - read a Char from stdin

-- MyChar.hs
main = do
  myChar <- getChar
  print myChar

-- In your shell

runhaskell MyChar.hs
a -- you enter a and press enter
'a' -- the program prints 'a'

getLine :: IO String - read a String from stdin, sans new line character
Prelude> getLine
Hello there!  -- user enters some text and presses enter
"Hello there!"

define read :: Read a => String -> a
- convert a String to a value

Prelude> read "1" :: Int
1
Prelude> read "1" :: Float
1.0
Prelude> read "True" :: Bool
True

Other, less common functions are:

- getContents :: IO String - returns all user input as a single string, which is read lazily as it is needed
- interact :: (String -> String) -> IO () - takes a function of type String->String as its argument. The entire input from the standard input device is passed to this function as its argument

Read IO online: https://riptutorial.com/haskell/topic/1904/io
Chapter 35: Lens

Introduction

**Lens** is a library for Haskell that provides lenses, isomorphisms, folds, traversals, getters and setters, which exposes a uniform interface for querying and manipulating arbitrary structures, not unlike Java's accessor and mutator concepts.

Remarks

### What is a Lens?

Lenses (and other optics) allow us to separate describing *how* we want to access some data from *what* we want to do with it. It is important to distinguish between the abstract notion of a lens and the concrete implementation. Understanding abstractly makes programming with **lens** much easier in the long run. There are many isomorphic representations of lenses so for this discussion we will avoid any concrete implementation discussion and instead give a high-level overview of the concepts.

### Focusing

An important concept in understanding abstractly is the notion of **focusing**. Important optics **focus** on a specific part of a larger data structure without forgetting about the larger context. For example, the **lens** \_1 focuses on the first element of a tuple but doesn't forget about what was in the second field.

Once we have focus, we can then talk about which operations we are allowed to perform with a lens. Given a **Lens s a** which when given a datatype of type **s** focuses on a specific **a**, we can either

1. Extract the **a** by forgetting about the additional context or
2. Replace the **a** by providing a new value

These correspond to the well-known **get** and **set** operations which are usually used to characterise a lens.

### Other Optics

We can talk about other optics in a similar fashion.

<table>
<thead>
<tr>
<th>Optic</th>
<th>Focuses on...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lens</td>
<td>One part of a product</td>
</tr>
</tbody>
</table>
Each optic focuses in a different way, as such, depending on which type of optic we have we can perform different operations.

## Composition

What's more, we can compose any of the two optics we have so-far discussed in order to specify complex data accesses. The four types of optics we have discussed form a lattice, the result of composing two optics together is their upper bound.

For example, if we compose together a lens and a prism, we get a traversal. The reason for this is that by their (vertical) composition, we first focus on one part of a product and then on one part of a sum. The result being an optic which focuses on precisely zero or one parts of our data which is a special case of a traversal. (This is also sometimes called an affine traversal).

## In Haskell

The reason for the popularity in Haskell is that there is a very succinct representation of optics. All optics are just functions of a certain form which can be composed together using function composition. This leads to a very light-weight embedding which makes it easy to integrate optics into your programs. Further to this, due to the particulars of the encoding, function composition also automatically computes the upper bound of two optics we compose. This means that we can reuse the same combinators for different optics without explicit casting.

## Examples

Manipulating tuples with Lens
Getting

("a", 1) ^. _1 -- returns "a"
("a", 1) ^. _2 -- returns 1

Setting

("a", 1) & _1 .~ "b" -- returns ("b", 1)

Modifying

("a", 1) & _2 %~ (+1) -- returns ("a", 2)

both Traversal

(1, 2) & both *~ 2 -- returns (2, 4)

Lenses for records

Simple record

{-# LANGUAGE TemplateHaskell #-}
import Control.Lens

data Point = Point {
  _x :: Float,
  _y :: Float
}
makesLenses "Point"

Lenses x and y are created.

let p = Point 5.0 6.0
p ^. x -- returns 5.0
set x 10 p -- returns Point { _x = 10.0, _y = 6.0 }
p & x +~ 1 -- returns Point { _x = 6.0, _y = 6.0 }

Managing records with repeating fields names

data Person = Person { _personName :: String }
makesFields "Person"

Creates a type class HasName, lens name for Person, and makes Person an instance of HasName. Subsequent records will be added to the class as well:

https://riptutorial.com/
The Template Haskell extension is required for `makeFields` to work. Technically, it's entirely possible to create the lenses made this way via other means, e.g. by hand.

**Stateful Lenses**

Lens operators have useful variants that operate in stateful contexts. They are obtained by replacing `~` with `=` in the operator name.

```
(+~) :: Num a => ASetter s t a a -> a -> s -> t
(+=) :: (MonadState s m, Num a) => ASetter' s a -> a -> m ()
```

Note: The stateful variants aren't expected to change the type, so they have the `Lens'` or `Simple Lens'` signatures.

### Getting rid of `&` chains

If lens-ful operations need to be chained, it often looks like this:

```
change :: A -> A
change a = a & lensA %~ operationA
               & lensB %~ operationB
               & lensC %~ operationC
```

This works thanks to the associativity of `&`. The stateful version is clearer, though.

```
change a = flip execState a $ do
            lensA %= operationA
            lensB %= operationB
            lensC %= operationC
```

If `lensX` is actually `id`, the whole operation can of course be executed directly by just lifting it with `modify`.

### Imperative code with structured state

Assuming this example state:

```
data Point = Point { _x :: Float, _y :: Float }
data Entity = Entity { _position :: Point, _direction :: Float }
data World = World { _entities :: [Entity] }
```

makeLenses ''Point
makeLenses ''Entity
makeLenses ''World

https://riptutorial.com/
We can write code that resembles classic imperative languages, while still allowing us to use benefits of Haskell:

```haskell
updateWorld :: MonadState World m => m ()
updateWorld = do
  -- move the first entity
  entities . ix 0 . position . x += 1

  -- do some operation on all of them
  entities . traversed . position %= \p -> p `pointAdd` ...

  -- or only on a subset
  entities . traversed . filtered (\e -> e ^. position.x > 100) %= ...
```

Writing a lens without Template Haskell

To demystify Template Haskell, suppose you have

```haskell
data Example a = Example { _foo :: Int, _bar :: a }
```

then

```haskell
makeLenses 'Example
```

produces (more or less)

```haskell
foo :: Lens' (Example a) Int
bar :: Lens (Example a) (Example b) a b
```

There's nothing particularly magical going on, though. You can write these yourself:

```haskell
foo :: Lens' (Example a) Int
  -- :: Functor f => (Int -> f Int) -> (Example a -> f (Example a)) ;; expand the alias
foo wrap (Example foo bar) = fmap (\newFoo -> Example newFoo bar) (wrap foo)

bar :: Lens (Example a) (Example b) a b
  -- :: Functor f => (a -> f b) -> (Example a -> f (Example b)) ;; expand the alias
bar wrap (Example foo bar) = fmap (\newBar -> Example foo newBar) (wrap bar)
```

Essentially, you want to "visit" your lens' "focus" with the `wrap` function and then rebuild the "entire" type.

Lens and Prism

A `Lens' s a` means that you can *always* find an `a` within any `s`. A `Prism' s a` means that you can *sometimes* find that `s` actually just *is* `a` but sometimes it's something else.

To be more clear, we have `_1 :: Lens' (a, b) a` because any tuple *always* has a first element. We have `_Just :: Prism' (Maybe a) a` because *sometimes* `Maybe a` is actually an `a` value wrapped in `Just` but *sometimes* it's `Nothing`. 

https://riptutorial.com/
With this intuition, some standard combinators can be interpreted parallel to one another:

- `view :: Lens' s a -> (s -> a)` "gets" the `a` out of the `s`.
- `set :: Lens' s a -> (a -> s -> s)` "sets" the `a` slot in `s`.
- `review :: Prism' s a -> (a -> s)` "realizes" that an `a` could be an `s`.
- `preview :: Prism' s a -> (s -> Maybe a)` "attempts" to turn an `s` into an `a`.

Another way to think about it is that a value of type `Lens' s a` demonstrates that `s` has the same structure as `(r, a)` for some unknown `r`. On the other hand, `Prism' s a` demonstrates that `s` has the same structure as `Either r a` for some `r`. We can write those four functions above with this knowledge:

```haskell
-- 'Lens' s a` is no longer supplied, instead we just *know* that `s ~ (r, a)`
view :: (r, a) -> a
view (r, a) = a

set :: a -> (r, a) -> (r, a)
set a (r, _) = (r, a)

-- 'Prism' s a` is no longer supplied, instead we just *know* that `s ~ Either r a`
review :: a -> Either r a
review a = Right a

preview :: Either r a -> Maybe a
preview (Left _) = Nothing
preview (Right a) = Just a
```

### Traversals

A `Traversal' s a` shows that `s` has 0-to-many `a`s inside of it.

```haskell
toListOf :: Traversal' s a -> (s -> [a])
```

Any type `t` which is `Traversable` automatically has that `traverse :: Traversal (t a) a`.

We can use a `Traversal` to set or map over all of these `a` values:

```haskell
> set (traverse 1) [1..10]
[1,1,1,1,1,1,1,1,1,1]

> over (traverse (+1)) [1..10]
[2,3,4,5,6,7,8,9,10,11]
```

A `f :: Lens' s a` says there’s exactly one `a` inside of `s`. A `g :: Prism' a b` says there are either 0 or 1 `b`s in `a`. Composing `f . g` gives us a `Traversal' s b` because following `f` and then `g` shows how there are 0-to-1 `b`s in `s`.

### Lenses compose

If you have a `f :: Lens' a b` and a `g :: Lens' b c` then `f . g` is a `Lens' a c` gotten by following `f`
first and then $g$. Notably:

- Lenses compose as functions (really they just are functions)
- If you think of the view functionality of Lens, it seems like data flows "left to right"—this might feel backwards to your normal intuition for function composition. On the other hand, it ought to feel natural if you think of .-notation like how it happens in OO languages.

More than just composing Lens with Lens, (.) can be used to compose nearly any "Lens-like" type together. It’s not always easy to see what the result is since the type becomes tougher to follow, but you can use the lens chart to figure it out. The composition $x \cdot y$ has the type of the least-upper-bound of the types of both $x$ and $y$ in that chart.

**Classy Lenses**

In addition to the standard `makeLenses` function for generating Lensess, Control.Lens.TH also offers the `makeClassy` function. `makeClassy` has the same type and works in essentially the same way as `makeLenses`, with one key difference. In addition to generating the standard lenses and traversals, if the type has no arguments, it will also create a class describing all the datatypes which possess the type as a field. For example

```haskell
data Foo = Foo { _fooX, _fooY :: Int }
makeClassy ''Foo
```

will create

```haskell
class HasFoo t where
  foo :: Simple Lens t Foo
instance HasFoo Foo where foo = id
fooX, fooY :: HasFoo t => Simple Lens t Int
```

**Fields with makeFields**

(This example copied from this StackOverflow answer)

Let’s say you have a number of different data types that all ought to have a lens with the same name, in this case capacity. The `makeFields` slice will create a class that accomplish this without namespace conflicts.

```haskell
{-# LANGUAGE FunctionalDependencies
  , MultiParamTypeClasses
  , TemplateHaskell
 #-}
module Foo
where
  import Control.Lens

data Foo
```
= Foo { fooCapacity :: Int }
deriving (Eq, Show)
$(makeFields "'Foo"

data Bar = Bar { barCapacity :: Double }
deriving (Eq, Show)
$(makeFields "'Bar"

Then in ghci:

*Foo
λ let f = Foo 3
|     b = Bar 7
|     
| b :: Bar
f :: Foo

*Foo
λ fooCapacity f
3
it :: Int

*Foo
λ barCapacity b
7.0
it :: Double

*Foo
λ f ^. capacity
3
it :: Int

*Foo
λ b ^. capacity
7.0
it :: Double

λ :info HasCapacity
class HasCapacity s a | s -> a where
capacity :: Lens' s a
-- Defined at Foo.hs:14:3
instance HasCapacity Foo Int -- Defined at Foo.hs:14:3
instance HasCapacity Bar Double -- Defined at Foo.hs:19:3

So what it's actually done is declared a class `HasCapacity s a`, where `capacity` is a `Lens`' from `s` to `a` (`a` is fixed once `s` is known). It figured out the name "capacity" by stripping off the (lowercased) name of the data type from the field; I find it pleasant not to have to use an underscore on either the field name or the lens name, since sometimes record syntax is actually what you want. You can use `makeFieldsWith` and the various `lensRules` to have some different options for calculating the lens names.

In case it helps, using ghci `-ddump-splices Foo.hs:

[1 of 1] Compiling Foo          ( Foo.hs, interpreted )
Foo.hs:14:3-18: Splicing declarations
makeFields "'Foo

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So the first splice made the class `HasCapacity` and added an instance for Foo; the second used the existing class and made an instance for Bar.

This also works if you import the `HasCapacity` class from another module; `makeFields` can add more instances to the existing class and spread your types out across multiple modules. But if you use it again in another module where you haven't imported the class, it'll make a new class (with the same name), and you'll have two separate overloaded capacity lenses that are not compatible.

Read Lens online: https://riptutorial.com/haskell/topic/891/lens
Chapter 36: List Comprehensions

Examples

Basic List Comprehensions

Haskell has list comprehensions, which are a lot like set comprehensions in math and similar implementations in imperative languages such as Python and JavaScript. At their most basic, list comprehensions take the following form.

```
[ x | x <- someList ]
```

For example

```
[ x | x <- [1..4] ]    -- [1,2,3,4]
```

Functions can be directly applied to x as well:

```
[ f x | x <- someList ]
```

This is equivalent to:

```
map f someList
```

Example:

```
[ x+1 | x <- [1..4]]    -- [2,3,4,5]
```

Patterns in Generator Expressions

However, x in the generator expression is not just variable, but can be any pattern. In cases of pattern mismatch the generated element is skipped over, and processing of the list continues with the next element, thus acting like a filter:

```
[x | Just x <- [Just 1, Nothing, Just 3]]     -- [1, 3]
```

A generator with a variable x in its pattern creates new scope containing all the expressions on its right, where x is defined to be the generated element.

This means that guards can be coded as

```
[ x | x <- [1..4], even x] ==
[ x | x <- [1..4], () <- [() | even x]] ==
[ x | x <- [1..4], () <- if even x then [()] else []]
```
Guards

Another feature of list comprehensions is guards, which also act as filters. Guards are Boolean expressions and appear on the right side of the bar in a list comprehension.

Their most basic use is

\[\{ x \mid p x \} \quad \text{===} \quad \text{if } p x \text{ then } \{ x \} \text{ else } \{ \}\]

Any variable used in a guard must appear on its left in the comprehension, or otherwise be in scope. So,

\[\{ f x \mid x \leftarrow \text{list}, \text{pred1} x y, \text{pred2} x \} \quad \text{-- `y` must be defined in outer scope}\]

which is equivalent to

\[
\text{map } f (\text{filter pred2 (filter } (\lambda x \to \text{pred1} x y) \text{ list})) \quad \text{-- or,} \\
\text{(} \$ \text{ list)} (\text{filter } (\`\text{pred1} \` y) \quad \text{>>>} \quad \text{filter pred2} \quad \text{>>>} \quad \text{map } f) \\
\text{list} \quad \gg\gg \quad (\lambda x \to \{ x \mid \text{pred1} x y \}) \quad \gg\gg \quad (\lambda x \to \{ x \mid \text{pred2} x \}) \quad \gg\gg \quad (\lambda x \to \{ f x \})
\]

(the \gg\gg \text{ operator is } \text{infixl 1}, \text{i.e. it associates (is parenthesized) to the left). Examples:}

\[\{ x \mid x \leftarrow \{1..4\}, \text{even } x \} \quad \text{-- } \{2,4\}\]

\[\{ x^2 + 1 \mid x \leftarrow \{1..100\}, \text{even } x \} \quad \text{-- map } (\lambda x \to x^2 + 1) \text{ (filter even } \{1..100\})\]

Nested Generators

List comprehensions can also draw elements from multiple lists, in which case the result will be the list of every possible combination of the two elements, as if the two lists were processed in the nested fashion. For example,

\[\{ (a,b) \mid a \leftarrow \{1,2,3\}, b \leftarrow \{'a','b'\}\} \]

\[\quad \text{-- } \{(1,'a'\}, \ (1,'b'), \ (2,'a'), \ (2,'b'), \ (3,'a'), \ (3,'b')\}\]

Parallel Comprehensions

With Parallel List Comprehensions language extension,

\[(x,y) \mid x \leftarrow xs \mid y \leftarrow ys\]

is equivalent to

\text{zip } xs \text{ ys}

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Example:

\[ (x, y) \mid x \leftarrow [1, 2, 3], y \leftarrow [10, 20] \]

\[ \{(1,10), (2,20)\} \]

**Local Bindings**

List comprehensions can introduce local bindings for variables to hold some interim values:

\[ (x, y) \mid x \leftarrow [1..4], \text{let } y = x^2 + 1, \text{ even } y \]

\[ \{(1,2), (3,10)\} \]

Same effect can be achieved with a trick,

\[ (x, y) \mid x \leftarrow [1..4], y \leftarrow [x^2+1], \text{ even } y \]

\[ \{(1,2), (3,10)\} \]

The `let` in list comprehensions is recursive, as usual. But generator bindings are not, which enables *shadowing*:

\[ [x \mid x \leftarrow [1..4], x \leftarrow [x^2+1], \text{ even } x] \]

\[ [2, 10] \]

**Do Notation**

Any list comprehension can be correspondingly coded with list monad’s do notation.

\[ [f x \mid x \leftarrow xs] \]

\[ f <$> xs \quad \text{do } \{ x \leftarrow xs ; \text{return } (f x) \} \]

\[ [f x \mid f \leftarrow fs, x \leftarrow xs] \]

\[ fs <*> xs \quad \text{do } \{ f \leftarrow fs ; x \leftarrow xs ; \text{return } (f x) \} \]

\[ [y \mid x \leftarrow xs, y \leftarrow f x] \]

\[ f =<< xs \quad \text{do } \{ x \leftarrow xs ; y \leftarrow f x ; \text{return } y \} \]

The guards can be handled using `Control.Monad.guard`:

\[ [x \mid x \leftarrow xs, \text{ even } x] \]

\[ \text{do } \{ x \leftarrow xs ; \text{guard } (\text{even } x) ; \text{return } x \} \]

**Read List Comprehensions online:** [https://riptutorial.com/haskell/topic/4970/list-comprehensions](https://riptutorial.com/haskell/topic/4970/list-comprehensions)

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Chapter 37: Lists

Syntax

1. empty list constructor

   \[
   \text{[]} :: [a]
   \]

2. non-empty list constructor

   \[
   (: \text{) :: a \to [a] \to [a]}
   \]

3. head - returns the first value of a list

   \[
   \text{head :: [a]} \to a
   \]

4. last - returns the last value of a list

   \[
   \text{last :: [a]} \to a
   \]

5. tail - returns a list without the first item

   \[
   \text{tail :: [a]} \to [a]
   \]

6. init - returns a list without the last item

   \[
   \text{init :: [a]} \to [a]
   \]

7. \(xs \text{ !! } i\) - return the element at an index i in list xs

   \[
   \text{(!!) :: Int \to [a] \to a}
   \]

8. take n xs - return new list containing n first elements of the list xs

   \[
   \text{take :: Int \to [a]} \to [a]
   \]

9. map :: (a \to b) \to [a] \to [b]

10. filter :: (a \to \text{Bool}) \to [a] \to [a]

11. ++ :: [a] \to [a]

12. concat :: [[a]] \to [a]

Remarks

1. The type \([a]\) is equivalent to \([] a\).
2. \([]\) constructs the empty list.
3. \([]\) in a function definition LHS, e.g. \(f [] = \ldots\), is the empty list pattern.
4. \(x:xs\) constructs a list where an element \(x\) is prepended to the list \(xs\)
5. \(f (x:xs) = \ldots\) is a pattern match for a non-empty list where \(x\) is the head and \(xs\) is the tail.
6. \(f (a:b:cs) = \ldots\)
and \( f(a:(b:cs)) = \ldots \) are the same. They are a pattern match for a list of at least two elements where the first element is \( a \), the second element is \( b \), and the rest of the list is \( cs \).

7. \( f((a:as):bs) = \ldots \) is NOT the same as \( f(a:(as:bs)) = \ldots \). The former is a pattern match for a non-empty list of lists, where \( a \) is the head of the head, \( as \) is the tail of the head, and \( bs \) is the tail.

8. \( f(x:] = \ldots \) and \( f[x = \ldots \) are the same. They are a pattern match for a list of exactly one element.

9. \( f(a:b:] = \ldots \) and \( f[a,b] = \ldots \) are the same. They are a pattern match for a list of exactly two elements.

10. \( f[a:b] = \ldots \) is a pattern match for a list of exactly one element where the element is also a list. \( a \) is the head of the element and \( b \) is the tail of the element.

11. \([a,b,c]\) is the same as \((a:b:c:])\). Standard list notation is just syntactic sugar for the (:) and [] constructors.

12. You can use all@(x:y:ys) in order to refer to the whole list as all (or any other name you choose) instead of repeating (x:y:ys) again.

Examples

List Literals

<table>
<thead>
<tr>
<th>list literal</th>
<th>notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>emptyList</td>
<td>[]</td>
</tr>
<tr>
<td>singletonList</td>
<td>[0]</td>
</tr>
<tr>
<td>listOfNums</td>
<td>[1, 2, 3]</td>
</tr>
<tr>
<td>listOfStrings</td>
<td>['A', 'B', 'C']</td>
</tr>
</tbody>
</table>

List Concatenation

| listA              | [1, 2, 3]                       |
| listB              | [4, 5, 6]                       |
| listAThenB         | listA ++ listB                  |
| ++ xs              | [] = xs                         |
| ++ []              | ys = ys                         |
| ++ (x:xs) ys       | x : (xs ++ ys)                  |

List basics

The type constructor for lists in the Haskell Prelude is []. The type declaration for a list holding values of type Int is written as follows:

```haskell
xs :: [Int] -- or equivalently, but less conveniently,
xs :: [] Int
```

Lists in Haskell are homogeneous sequences, which is to say that all elements must be of the
same type. Unlike tuples, list type is not affected by length:

```
[1,2,3] :: [Int]
[1,2,3,4] :: [Int]
```

Lists are constructed using **two constructors**:

- `[]` constructs an empty list.
- `(:)`, pronounced "cons", prepends elements to a list. Consing `x` (a value of type `a`) onto `xs` (a list of values of the same type `a`) creates a new list, whose **head** (the first element) is `x`, and **tail** (the rest of the elements) is `xs`.

We can define simple lists as follows:

```
ys :: [a]
y = []

xs :: [Int]
x = 12 : (99 : (37 : []))
-- or = 12 : 99 : 37 : []       -- (:) is right-associative)
-- or = [12, 99, 37]          -- (syntactic sugar for lists)
```

Note that `(++)`, which can be used to build lists is defined recursively in terms of `(:)` and `[]`.

### Processing lists

To process lists, we can simply pattern match on the constructors of the list type:

```
listSum :: [Int] -> Int
listSum [] = 0
listSum (x:xs) = x + listSum xs
```

We can match more values by specifying a more elaborate pattern:

```
sumTwoPer :: [Int] -> Int
sumTwoPer [] = 0
sumTwoPer (x1:x2:xs) = x1 + x2 + sumTwoPer xs
sumTwoPer (x:xs) = x + sumTwoPer xs
```

Note that in the above example, we had to provide a more exhaustive pattern match to handle cases where an odd length list is given as an argument.

The Haskell Prelude defines many built-ins for handling lists, like `map`, `filter`, etc.. Where possible, you should use these instead of writing your own recursive functions.

### Accessing elements in lists

Access the `n`th element of a list (zero-based):

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list = [1 .. 10]

firstElement = list !! 0  -- 1

Note that `!!` is a partial function, so certain inputs produce errors:

list !! (-1)     -- *** Exception: Prelude.!!: negative index
list !! 1000     -- *** Exception: Prelude.!!: index too large

There's also `Data.List.genericIndex`, an overloaded version of `!!`, which accepts any `Integral` value as the index.

import Data.List (genericIndex)

list `genericIndex` 4              -- 5

When implemented as singly-linked lists, these operations take $O(n)$ time. If you frequently access elements by index, it's probably better to use `Data.Vector` (from the `vector` package) or other data structures.

Ranges

Creating a list from 1 to 10 is simple using range notation:

[1..10]   -- [1,2,3,4,5,6,7,8,9,10]

To specify a step, add a comma and the next element after the start element:

[1,3..10]  -- [1,3,5,7,9]

Note that Haskell always takes the step as the arithmetic difference between terms, and that you cannot specify more than the first two elements and the upper bound:

[1,3,5..10] -- error
[1,3,9..20] -- error

To generate a range in descending order, always specify the negative step:

[5..1]     -- []
[5,4..1]   -- [5,4,3,2,1]

Because Haskell is non-strict, the elements of the list are evaluated only if they are needed, which allows us to use infinite lists. `[1..]` is an infinite list starting from 1. This list can be bound to a variable or passed as a function argument:

take 5 [1..]  -- returns [1,2,3,4,5] even though [1..] is infinite
Be careful when using ranges with floating-point values, because it accepts spill-overs up to half-delta, to fend off rounding issues:

```
[1.0,1.5..2.4]    -- [1.0,1.5,2.0,2.5] , though 2.5 > 2.4
[1.0,1.1..1.2]    -- [1.0,1.1,1.2000000000000002] , though 1.2000000000000002 > 1.2
```

Ranges work not just with numbers but with any type that implements `Enum` typeclass. Given some enumerable variables `a`, `b`, `c`, the range syntax is equivalent to calling these `Enum` methods:

```
[a..]   == enumFrom a
[a..c]  == enumFromTo a c
[a,b..] == enumFromThen a b
[a,b..c]== enumFromThenTo a b c
```

For example, with `Bool` it’s

```
[False ..]  -- [False,True]
```

Notice the space after `False`, to prevent this to be parsed as a module name qualification (i.e. `False..` would be parsed as `..` from a module `False`).

**Basic Functions on Lists**

```
head [1..10]   --  1
last [1..20]   --  20
tail [1..5]    --  [2, 3, 4, 5]
init [1..5]    --  [1, 2, 3, 4]
length [1..10] --  10
reverse [1..10] --  [10, 9 .. 1]
take 5 [1..5]  --  [1, 2, 3, 4, 5]
drop 5 [1..10]  --  [6, 7, 8, 9, 10]
concat [[1,2], [], [4]]  --  [1,2,4]
```

**foldl**

This is how the left fold is implemented. Notice how the order of the arguments in the step function is flipped compared to `foldr` (the right fold):

```
foldl :: (b -> a -> b) -> b -> [a] -> b
foldl f acc []     = acc
foldl f acc (x:xs) = foldl f (f acc x) xs  -- = foldl f (acc `f` x) xs
```

The left fold, `foldl`, associates to the left. That is:
foldl (+) 0 [1, 2, 3]     -- is equivalent to ((0 + 1) + 2) + 3

The reason is that foldl is evaluated like this (look at foldl's inductive step):

foldl (+) 0 [1, 2, 3]                        -- foldl (+) 0 [ 1, 2, 3 ]
foldl (+) ((+) 0 1) [2, 3]                   -- foldl (+) (0 + 1) [ 2, 3 ]
foldl (+) ((+) ((+) 0 1) 2) [3]              -- foldl (+) (((0 + 1) + 2) [ 3 ]
foldl (+) (((+) ((+) (+) 0 1) 2) 3) []      -- foldl (+) (((0 + 1) + 2) + 3) []
((+) ((+) ((+) 0 1) 2) 3)                      -- (((0 + 1) + 2) + 3)

The last line is equivalent to ((0 + 1) + 2) + 3. This is because (f a b) is the same as (a `f` b) in general, and so ((+) 0 1) is the same as (0 + 1) in particular.

foldr

This is how the right fold is implemented:

foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z []     = z
foldr f z (x:xs) = f x (foldr f z xs)              -- = x `f` foldr f z xs

The right fold, foldr, associates to the right. That is:

foldr (+) 0 [1, 2, 3]     -- is equivalent to 1 + (2 + (3 + 0))

The reason is that foldr is evaluated like this (look at the inductive step of foldr):

foldr (+) 0 [1, 2, 3]                        -- foldr (+) 0 [1,2,3]
(+) 1 (foldr (+) 0 [2, 3])                   -- 1 + foldr (+) 0 [2,3]
(+) 1 ((+) 2 (foldr (+) 0 [3]))              -- 1 + (2 + foldr (+) 0 [3])
(+) 1 ((+) 2 ((+) 3 (foldr (+) 0 [])))      -- 1 + (2 + (3 + foldr (+) 0 []))
(+) 1 ((+) 2 ((+) 3 0))                      -- 1 + (2 + (3 + 0 )

The last line is equivalent to 1 + (2 + (3 + 0)), because ((+) 3 0) is the same as (3 + 0).

Transforming with `map`

Often we wish to convert, or transform the contents of a collection (a list, or something traversable). In Haskell we use map:

-- Simple add 1
map (+ 1) [1,2,3]
[2,3,4]

map odd [1,2,3]
[True,False,True]

data Gender = Male | Female deriving Show
data Person = Person String Gender Int deriving Show

-- Extract just the age from a list of people
map (\(\text{Person n g a} \rightarrow a\)) [(\text{Person "Alex" Male 31}), (\text{Person "Ellie" Female 29})]

\[31,29\]

Filtering with `filter`

Given a list:

\[
\text{li} = [1,2,3,4,5]
\]

we can filter a list with a predicate using \(\text{filter} : (a \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [a]\):

\[
\begin{align*}
\text{filter} (== 1) \text{li} & \quad -- [1] \\
\text{filter} \text{ (even)} \text{li} & \quad -- [2,4] \\
\text{filter} \text{ (odd)} \text{li} & \quad -- [1,3,5] \\
\end{align*}
\]

-- Something slightly more complicated
\[
\text{comfy i} = \text{notTooLarge && isEven}
\]

where
\[
\begin{align*}
\text{notTooLarge} & = (i + 1) < 5 \\
\text{isEven} & = \text{even i}
\end{align*}
\]

\[
\text{filter} \text{ comfy li} \quad -- [2]
\]

Of course it's not just about numbers:

data Gender = Male | Female deriving Show
data Person = Person String Gender Int deriving Show

\[
\begin{align*}
\text{onlyLadies} :: [\text{Person}] \rightarrow \text{Person} \\
\text{onlyLadies} \text{x} & = \text{filter isFemale x}
\end{align*}
\]

where
\[
\begin{align*}
\text{isFemale} (\text{Person _ Female _}) & = \text{True} \\
\text{isFemale _} & = \text{False}
\end{align*}
\]

\[
\text{onlyLadies} [(\text{Person "Alex" Male 31}), (\text{Person "Ellie" Female 29})] \\
= [\text{Person "Ellie" Female 29}]
\]

Zipping and Unzipping Lists

zip takes two lists and returns a list of corresponding pairs:

\[
\begin{align*}
\text{zip} \ [\text{]} \ _ & \quad = \ [\text{]} \\
\text{zip} \ _ \ [\text{]} & \quad = \ [\text{]} \\
\text{zip} \ (a:\text{as}) \ (b:\text{bs}) & = (a,b) : \text{zip as bs}
\end{align*}
\]

> \text{zip} \ [1,3,5] \ [2,4,6] \\
> [(1,2),(3,4),(5,6)]

Zipping two lists with a function:

\[
\text{zipWith} \ f \ [\text{]} \ _ \quad = \ [\text{]}
\]
zipWith \( f \_ \_ \) \([\] \) \([\] \) = \([\] \)
zipWith \( f \) \( \text{a:as} \) \( \text{b:bs} \) = \( f \text{a b} \) : zipWith \( f \text{as bs} \)

\[
> \text{zipWith (+) [1,3,5] [2,4,6]} \\
> [3,7,11]
\]

**Unzipping a list:**

\[
\text{unzip = foldr (\((a,b) \sim (\text{as,bs}) \rightarrow (\text{a:as,b:bs})\)) ([],[])}
\]

\[
> \text{unzip [(1,2),(3,4),(5,6)]} \\
> ([1,3,5],[2,4,6])
\]

*Read Lists online: [https://riptutorial.com/haskell/topic/2281/lists](https://riptutorial.com/haskell/topic/2281/lists)*
Chapter 38: Logging

Introduction

Logging in Haskell is achieved usually through functions in the `IO` monad, and so is limited to non-pure functions or "IO actions".

There are several ways to log information in a Haskell program: from `putStrLn` (or `print`), to libraries such as `hslogger` or through `Debug.Trace`.

Examples

Logging with hslogger

The `hslogger` module provides a similar API to Python's `logging` framework, and supports hierarchically named loggers, levels and redirection to handles outside of `stdout` and `stderr`.

By default, all messages of level `WARNING` and above are sent to `stderr` and all other log levels are ignored.

```haskell
import System.Log.Logger (Priority (DEBUG), debugM, infoM, setLevel, updateGlobalLogger, warningM)

main = do
  debugM "MyProgram.main" "This won't be seen"
  infoM "MyProgram.main" "This won't be seen either"
  warningM "MyProgram.main" "This will be seen"

We can set the level of a logger by its name using `updateGlobalLogger`:

```haskell
updateGlobalLogger "MyProgram.main" (setLevel DEBUG)

debugM "MyProgram.main" "This will now be seen"
```

Each Logger has a name, and they are arranged hierarchically, so `MyProgram` is a parent of `MyParent.Module`.

Read Logging online: https://riptutorial.com/haskell/topic/9628/logging
Chapter 39: Modules

Syntax

- module Name where -- export all names declared in this file
- module Name (functionOne, Type (..)) where -- export only functionOne, Type, and Type's constructors
- import Module -- import all of Module's exported names
- import qualified Module as MN -- qualified import
- import Module (justThisFunction) -- import only certain names from a module
- import Module hiding (functionName, Type) -- import all names from a module except functionName and Type

Remarks

Haskell has support for modules:

- a module can export all, or a subset of its member types & functions
- a module can "re-export" names it imported from other modules

On the consumer end of a module, one can:

- import all, or a subset of module members
- hide imports of a particular member or set of members

haskell.org has a great chapter on module definition.

Examples

Defining Your Own Module

If we have a file called Business.hs, we can define a Business module that can be import-ed, like so:

```haskell
module Business (Person (..), employees) where
  -- ^ Export the Person type and all its constructors and field names
  -- ^ Export the employees function

  -- begin types, function definitions, etc
```

A deeper hierarchy is of course possible; see the Hierarchical module names example.
Exporting Constructors

To export the type and all its constructors, one must use the following syntax:

```haskell
module X (Person(..)) where
```

So, for the following top-level definitions in a file called `People.hs`:

```haskell
data Person = Friend String | Foe deriving (Show, Eq, Ord)

isFoe Foe = True
isFoe _   = False
```

This module declaration at the top:

```haskell
module People (Person(..)) where
```

would only export `Person` and its constructors `Friend` and `Foe`.

If the export list following the module keyword is omitted, all of the names bound at the top level of the module would be exported:

```haskell
module People where
```

would export `Person`, its constructors, and the `isFoe` function.

Importing Specific Members of a Module

Haskell supports importing a subset of items from a module.

```haskell
import qualified Data.Stream (map) as D
```

would only import `map` from `Data.Stream`, and calls to this function would require `D.:

```haskell
D.map odd [1..]
```

otherwise the compiler will try to use Prelude's `map` function.

Hiding Imports

Prelude often defines functions whose names are used elsewhere. Not hiding such imports (or using qualified imports where clashes occur) will cause compilation errors.

`Data.Stream` defines functions named `map`, `head` and `tail` which normally clashes with those defined in Prelude. We can hide those imports from Prelude using `hiding`:

```haskell
import Data.Stream -- everything from Data.Stream
import Prelude hiding (map, head, tail, scan, foldl, foldr, filter, dropWhile, take) -- etc
```
In reality, it would require too much code to hide Prelude clashes like this, so you would in fact use a qualified import of `Data.Stream` instead.

**Qualifying Imports**

When multiple modules define the same functions by name, the compiler will complain. In such cases (or to improve readability), we can use a qualified import:

```haskell
import qualified Data.Stream as D
```

Now we can prevent ambiguity compiler errors when we use `map`, which is defined in Prelude and Data.Stream:

```haskell
map (== 1) [1,2,3] -- will use Prelude.map
D.map (odd) (fromList [1..]) -- will use Data.Stream.map
```

It is also possible to import a module with only the clashing names being qualified via `import Data.Text as T`, which allows one to have `Text` instead of `T.Text` etc.

**Hierarchical module names**

The names of modules follow the filesystem's hierarchical structure. With the following file layout:

```
Foo/
  ├── Baz/
  │   └── Quux.hs
  └── Bar.hs
Foo.hs
Bar.hs
```

the module headers would look like this:

```
-- file Foo.hs
module Foo where

-- file Bar.hs
module Bar where

-- file Foo/Bar.hs
module Foo.Bar where

-- file Foo/Baz/Quux.hs
module Foo.Baz.Quux where
```

Note that:

- the module name is based on the path of the file declaring the module
- Folders may share a name with a module, which gives a naturally hierarchical naming structure to modules

Read Modules online: https://riptutorial.com/haskell/topic/5234/modules
Chapter 40: Monad Transformers

Examples

A monadic counter

An example on how to compose the reader, writer, and state monad using monad transformers. The source code can be found in this repository.

We want to implement a counter, that increments its value by a given constant.

We start by defining some types, and functions:

```haskell
newtype Counter = MkCounter {cValue :: Int}
deriving (Show)

-- | 'inc c n' increments the counter by 'n' units.
inc :: Counter -> Int -> Counter
inc (MkCounter c) n = MkCounter (c + n)
```

Assume we want to carry out the following computation using the counter:

- set the counter to 0
- set the increment constant to 3
- increment the counter 3 times
- set the increment constant to 5
- increment the counter 2 times

The state monad provides abstractions for passing state around. We can make use of the state monad, and define our increment function as a state transformer.

```haskell
-- | CounterS is a monad.
type CounterS = State Counter

-- | Increment the counter by 'n' units.
incS :: CounterS ()
incS n = modify (\c -> inc c n)
```

This already enables us to express a computation in a more clear and succinct way:

```haskell
-- | The computation we want to run, with the state monad.
mComputationS :: CounterS ()
mComputationS = do
  incS 3
  incS 3
  incS 3
  incS 5
  incS 5
```

But we still have to pass the increment constant at each invocation. We would like to avoid this.
Adding an environment

The **reader monad** provides a convenient way to pass an environment around. This monad is used in functional programming to perform what in the OO world is known as **dependency injection**.

In its simplest version, the reader monad requires two types:

- the type of the value being read (i.e. our environment, \( r \) below),
- the value returned by the reader monad (\( a \) below).

\[
\text{Reader } r \ a
\]

However, we need to make use of the state monad as well. Thus, we need to use the **ReaderT** transformer:

\[
\text{newtype ReaderT } r \ m \ a :: \ast \to \ast \to \ast \to \ast
\]

Using **ReaderT**, we can define our counter with environment and state as follows:

\[
\text{type CounterRS } = \text{ReaderT } \text{Int } \text{CounterS}
\]

We define an **incR** function that takes the increment constant from the environment (using **ask**), and to define our increment function in terms of our **CounterS** monad we make use of the **lift** function (which belongs to the **monad transformer** class).

\[
\begin{aligned}
\text{-- | Increment the counter by the amount of units specified by the environment.} \\
\text{incR :: CounterRS } () \\
\text{incR }\equiv\text{ ask } \gg\gg \text{ lift } \cdot \text{incS}
\end{aligned}
\]

Using the reader monad we can define our computation as follows:

\[
\begin{aligned}
\text{-- | The computation we want to run, using reader and state monads.} \\
\text{mComputationRS :: CounterRS } () \\
m\text{ComputationRS } = \text{ do} \\
\text{local } (\text{const } 3) \$ \text{ do} \\
\text{incR} \\
\text{incR} \\
\text{incR} \\
\text{local } (\text{const } 5) \$ \text{ do} \\
\text{incR} \\
\text{incR}
\end{aligned}
\]

The requirements changed: we need logging!

Now assume that we want to add logging to our computation, so that we can see the evolution of our counter in time.
We also have a monad to perform this task, the writer monad. As with the reader monad, since we are composing them, we need to make use of the reader monad transformer:

```
newtype WriterT w m a :: * -> (* -> *) -> * -> *
```

Here \( w \) represents the type of the output to accumulate (which has to be a monoid, which allow us to accumulate this value), \( m \) is the inner monad, and \( a \) the type of the computation.

We can then define our counter with logging, environment, and state as follows:

```
type CounterWRS = WriterT [Int] CounterRS
```

And making use of `lift` we can define the version of the increment function which logs the value of the counter after each increment:

```
incW :: CounterWRS ()
incW = lift incR >> get >>= tell . (:[[]]) . cValue
```

Now the computation that contains logging can be written as follows:

```
mComputationWRS :: CounterWRS ()
mComputationWRS = do
  local (const 3) $ do
    incW
    incW
    incW
  local (const 5) $ do
    incW
    incW
```

---

**Doing everything in one go**

This example intended to show monad transformers at work. However, we can achieve the same effect by composing all the aspects (environment, state, and logging) in a single increment operation.

To do this we make use of type-constraints:

```
inc' :: (MonadReader Int m, MonadState Counter m, MonadWriter [Int] m) => m ()
inc' = ask >>= modify . (flip inc) >> get >>= tell . (:[[]]) . cValue
```

Here we arrive at a solution that will work for any monad that satisfies the constraints above. The computation function is defined thus with type:

```
mComputation' :: (MonadReader Int m, MonadState Counter m, MonadWriter [Int] m) => m ()
```

since in its body we make use of `inc'`. 

https://riptutorial.com/
We could run this computation, in the ghci REPL for instance, as follows:

```haskell
runState ( runReaderT ( runWriterT mComputation' ) 15 ) (MkCounter 0)
```

Read Monad Transformers online: https://riptutorial.com/haskell/topic/7752/monad-transformers
Chapter 41: Monads

Introduction

A monad is a data type of composable actions. Monad is the class of type constructors whose values represent such actions. Perhaps IO is the most recognizable one: a value of IO a is a "recipe for retrieving an a value from the real world".

We say a type constructor m (such as [] or Maybe) forms a monad if there is an instance Monad m satisfying certain laws about composition of actions. We can then reason about m a as an "action whose result has type a".

Examples

The Maybe monad

Maybe is used to represent possibly empty values - similar to null in other languages. Usually it is used as the output type of functions that can fail in some way.

Consider the following function:

```haskell
halve :: Int -> Maybe Int
halve x
| even x = Just (x `div` 2)
| odd x = Nothing
```

Think of `halve` as an action, depending on an Int, that tries to halve the integer, failing if it is odd.

How do we `halve` an integer three times?

```haskell
takeOneEighth :: Int -> Maybe Int            -- (after you read the 'do' sub-section:)
takeOneEighth x =
  case halve x of
    Nothing -> Nothing
    Just oneHalf ->
      case halve oneHalf of
        Nothing -> Nothing
        Just oneQuarter ->
          case halve oneQuarter of
            Nothing -> Nothing
            Just oneEighth ->
              Just oneEighth
```

- `takeOneEighth` is a sequence of three `halve` steps chained together.
- If a `halve` step fails, we want the whole composition `takeOneEighth` to fail.
- If a `halve` step succeeds, we want to pipe its result forward.

instance Monad Maybe where
(>>=) :: Maybe a -> (a -> Maybe b) -> Maybe b
Nothing >>= f  = Nothing                         -- infixl 1 >>=
Just x >>= f = Just (f x)                       -- also, f =<< m = m >>= f

-- return :: a -> Maybe a
return x   = Just x

and now we can write:

takeOneEighth :: Int -> Maybe Int
takeOneEighth x = halve x >>= halve >>= halve             -- or,
                   -- return x >>= halve >>= halve              -- which is parsed as
                   -- ((return x) >>= halve) >>= halve          -- which can also be written as
                   -- (halve =<<) . (halve =<<) . (halve =<<) $ return x  -- or, equivalently, as
                   -- halve <=< halve <=< halve $ x

Kleisli composition <=< is defined as (g <=< f) x = g =<< f x, or equivalently as (f >>= g) x = f x
>>= g. With it the above definition becomes just

takeOneEighth :: Int -> Maybe Int
takeOneEighth = halve <=< halve <=< halve              -- infixr 1 <=<
                   -- or, equivalently,
                   -- halve >>=> halve >>=> halve               -- infixr 1 >>=>

There are three monad laws that should be obeyed by every monad, that is every type which is an
instance of the Monad typeclass:

1.  return x >>= f  =  f x
2.    m >>= return  =  m
3. (m >>= g) >>= h  =  m >>= (
y -> g y >>= h)

where m is a monad, f has type a -> m b and g has type b -> m c.

Or equivalently, using the >>= Kleisli composition operator defined above:

1. return >>= g = g
2. f >>= return = f
3. (f >>= g) >>= h = f >>= (g >>= h)

Obeying these laws makes it a lot easier to reason about the monad, because it guarantees that
using monadic functions and composing them behaves in a reasonable way, similar to other
monads.

Let's check if the Maybe monad obeys the three monad laws.

1. The left identity law  - return x >>= f = f x

return z >>= f  
= (Just z) >>= f 
= f z
2. The right identity law - \( m >>= \text{return} = m \)

- Just data constructor

```haskell
Just z >>= \text{return} = \text{return} z = Just z
```

- Nothing data constructor

```haskell
Nothing >>= \text{return} = \text{Nothing}
```

3. The associativity law - \((m >>= f) >>= g = m >>= (\lambda x \rightarrow f \ x >>= g)\)

- Just data constructor

```haskell
-- Left-hand side
((\text{Just} \ z) >>= f) >>= g = f \ z >>= g

-- Right-hand side
(\text{Just} \ z) >>= (\lambda x \rightarrow f \ x >>= g)
(\lambda x \rightarrow f \ x >>= g) z = f \ z >>= g
```

- Nothing data constructor

```haskell
-- Left-hand side
(\text{Nothing} >>= f) >>= g = \text{Nothing} >>= g = \text{Nothing}

-- Right-hand side
\text{Nothing} >>= (\lambda x \rightarrow f \ x >>= g) = \text{Nothing}
```

**IO monad**

There is no way to get a value of type \( a \) out of an expression of type \( \text{IO} \ a \) and there shouldn't be. This is actually a large part of why monads are used to model \( \text{IO} \).

An expression of type \( \text{IO} \ a \) can be thought of as representing an action that can interact with the real world and, if executed, would result in something of type \( a \). For example, the function `getLine :: \text{IO} \ \text{String}` from the prelude doesn't mean that underneath `getLine` there is some specific string that I can extract - it means that `getLine` represents the action of getting a line from standard input.

Not surprisingly, `main :: \text{IO} ()` since a Haskell program does represent a computation/action that interacts with the real world.

The things you can do to expressions of type \( \text{IO} \ a \) because \( \text{IO} \) is a monad:
• Sequence two actions using `(>>)` to produce a new action that executes the first action, discards whatever value it produced, and then executes the second action.

```haskell
gputStrLn "Hello" >> putStrLn "World"
```

• Sometimes you don’t want to discard the value that was produced in the first action - you’d actually like it to be fed into a second action. For that, we have `(>>=)`. For `IO`, it has type `(>>=) :: IO a -> (a -> IO b) -> IO b`.

```haskell
getLine >>= putStrLn
```

• Take a normal value and convert it into an action which just immediately returns the value you gave it. This function is less obviously useful until you start using `do` notation.

```haskell
return 5
```

More from the Haskell Wiki on the IO monad [here](https://riptutorial.com/).

### List Monad

The lists form a monad. They have a monad instantiation equivalent to this one:

```haskell
instance Monad [] where
  return x = [x]
  xs >>= f = concat (map f xs)
```

We can use them to emulate non-determinism in our computations. When we use `xs >>= f`, the function `f :: a -> [b]` is mapped over the list `xs`, obtaining a list of lists of results of each application of `f` over each element of `xs`, and all the lists of results are then concatenated into one list of all the results. As an example, we compute a sum of two non-deterministic numbers using `do-notation`, the sum being represented by list of sums of all pairs of integers from two lists, each list representing all possible values of a non-deterministic number:

```haskell
sumnd xs ys = do
  x <- xs
  y <- ys
  return (x + y)
```

Or equivalently, using `liftM2` in `Control.Monad`:

```haskell
sumnd = liftM2 (+)
```

we obtain:

```haskell
> sumnd [1,2,3] [0,10]
```
Monad as a Subclass of Applicative

As of GHC 7.10, Applicative is a superclass of Monad (i.e., every type which is a Monad must also be an Applicative). All the methods of Applicative (pure, <*> ) can be implemented in terms of methods of Monad (return, >>=).

It is obvious that pure and return serve equivalent purposes, so pure = return. The definition for <*> is too relatively clear:

\[
mf <*> mx = do \{ f <- mf; x <- mx; return (f x) \} \\
\quad -- = mf >>= (\f -> mx >>= (\x -> return (f x)))) \\
\quad -- = [r | f <- mf, x <- mx, r <- return (f x)] -- with MonadComprehensions \\
\quad -- = [f x | f <- mf, x <- mx]
\]

This function is defined as ap in the standard libraries.

Thus if you have already defined an instance of Monad for a type, you effectively can get an instance of Applicative for it "for free" by defining

```haskell
instance Applicative < type > where
  pure = return
  <*> = ap
```

As with the monad laws, these equivalencies are not enforced, but developers should ensure that they are always upheld.

No general way to extract value from a monadic computation

You can wrap values into actions and pipe the result of one computation into another:

```haskell
return :: Monad m => a -> m a
(>>=) :: Monad m => m a -> (a -> m b) -> m b
```

However, the definition of a Monad doesn’t guarantee the existence of a function of type \( \text{Monad } m \rightarrow m \ a \rightarrow a \).

That means there is, in general, no way to extract a value from a computation (i.e. “unwrap” it). This is the case for many instances:

```haskell
extract :: Maybe a -> a
extract (Just x) = x -- Sure, this works, but...
extract Nothing = undefined -- We can’t extract a value from failure.
```

Specifically, there is no function \( \text{IO } a \rightarrow a \), which often confuses beginners; see this example.

do-notation

https://riptutorial.com/
do-notation is syntactic sugar for monads. Here are the rules:

\[
\begin{align*}
do x <- mx & \quad \text{is equivalent to} & \quad do x <- mx \\
y <- my & \quad \text{is equivalent to} & \quad do y <- my \\
... & \quad & \\
\end{align*}
\]

\[
\begin{align*}
do let a = b & \quad \text{is equivalent to} & \quad do a = b \text{ in} \\
... & \quad & \text{do } ... \\
\end{align*}
\]

\[
\begin{align*}
do m & \quad \text{is equivalent to} & \quad m >> ( \\
e & \quad \text{is equivalent to} & \quad e) \\
\end{align*}
\]

\[
\begin{align*}
do x <- m & \quad \text{is equivalent to} & \quad m >>= (\lambda x -> \\
e & \quad \text{is equivalent to} & \quad e) \\
\end{align*}
\]

\[
\begin{align*}
do m & \quad \text{is equivalent to} & \quad m \\
\end{align*}
\]

For example, these definitions are equivalent:

```haskell
example :: IO Integer
example =
do putStrLn "What's your name?" >>
   getLine >>= (\name -> putStrLn ("Hello, " ++ name ++ ".") >>
                  putStrLn "What should we return?" >>
                  getLine >>= (\line ->
                                 let n = (read line :: Integer) in
                                 return (n + n))))
```

```haskell
example :: IO Integer
example = do
  putStrLn "What's your name?"
n = getLine
  putStrLn ("Hello, " ++ n ++ ".")
prior putStrLn "What should we return?"
  line = getLine
  n = (read line :: Integer)
return (n + n)
```

Definition of Monad

```haskell
class Monad m where
  return :: a -> m a
  (>>=) :: m a -> (a -> m b) -> m b
```

The most important function for dealing with monads is the **bind operator >>=**: 

```haskell
(>>=) :: m a -> (a -> m b) -> m b
```

- Think of \( m a \) as "an action with an \( a \) result".
- Think of \( a -> m b \) as “an action (depending on an \( a \) parameter) with a \( b \) result.”.
sequences two actions together by piping the result from the first action to the second.

The other function defined by Monad is:

\[ \text{return} : a \rightarrow m a \]

Its name is unfortunate: this return has nothing to do with the return keyword found in imperative programming languages.

\[ \text{return } x \text{ is the trivial action yielding } x \text{ as its result. (It is trivial in the following sense:)} \]

\[
\begin{align*}
\text{return } x >> \text{f} & \quad = \quad f x \quad \quad \text{-- "left identity" monad law} \\
\text{x >> return} & \quad = \quad \text{x} \quad \quad \text{-- "right identity" monad law}
\end{align*}
\]

Read Monads online: https://riptutorial.com/haskell/topic/2968/monads
Chapter 42: Monoid

Examples

An instance of Monoid for lists

```haskell
instance Monoid [a] where
  mempty  = []
  mappend = (++)
```

Checking the Monoid laws for this instance:

```haskell
mempty `mappend` x = x   <->   [] ++ xs = xs  -- prepending an empty list is a no-op
x `mappend` mempty = x   <->   xs ++ [] = xs  -- appending an empty list is a no-op
x `mappend` (y `mappend` z) = (x `mappend` y) `mappend` z
  <->   xs ++ (ys ++ zs) = (xs ++ ys) ++ zs           -- appending lists is associative
```

Collapsing a list of Monoids into a single value

```haskell
mconcat :: [a] -> a
```

is another method of the Monoid typeclass:

```haskell
ghci> mconcat [Sum 1, Sum 2, Sum 3]
Sum {getSum = 6}
ghci> mconcat ["concat", "enate"]
" concatenate"
```

Its default definition is `mconcat = foldr mappend mempty`.

Numeric Monoids

Numbers are monoidal in two ways: addition with 0 as the unit, and multiplication with 1 as the unit. Both are equally valid and useful in different circumstances. So rather than choose a preferred instance for numbers, there are two newtypes, `Sum` and `Product` to tag them for the different functionality.

```haskell
newtype Sum n = Sum { getSum :: n }

instance Num n => Monoid (Sum n) where
  mempty = Sum 0
  Sum x `mappend` Sum y = Sum (x + y)

newtype Product n = Product { getProduct :: n }

instance Num n => Monoid (Product n) where
  mempty = Product 1
  Product x `mappend` Product y = Product (x * y)
```
This effectively allows for the developer to choose which functionality to use by wrapping the value in the appropriate `newtype`.

| Sum 3     <> Sum 5     == Sum 8        |
|----------|----------------------|
| Product 3 <> Product 5 == Product 15 |

An instance of Monoid for ()

() is a Monoid. Since there is only one value of type (), there’s only one thing `mempty` and `mappend` could do:

```haskell
instance Monoid () where
  mempty = ()
  () `mappend` () = ()
```

Read Monoid online: https://riptutorial.com/haskell/topic/2211/monoid
Chapter 43: Optimization

Examples

Compiling your Program for Profiling

The GHC compiler has mature support for compiling with profiling annotations.

Using the -prof and -fprof-auto flags when compiling will add support to your binary for profiling flags for use at runtime.

Suppose we have this program:

```haskell
main = print (fib 30)
fib n = if n < 2 then 1 else fib (n-1) + fib (n-2)
```

Compiled it like so:

```
ghc -prof -fprof-auto -rtsopts Main.hs
```

Then ran it with runtime system options for profiling:

```
./Main +RTS -p
```

We will see a `main.prof` file created post execution (once the program has exited), and this will give us all sorts of profiling information such as cost centers which gives us a breakdown of the cost associated with running the various parts of the code:

```
Main +RTS -p -RTS

total time = 0.68 secs  (34 ticks @ 20 ms)
total alloc = 204,677,844 bytes  (excludes profiling overheads)

COST CENTRE MODULE  %time %alloc
fib         Main    100.0  100.0

individual inherited
COST CENTRE MODULE                  no.     entries  %time %alloc   %time %alloc
MAIN        MAIN                    102           0    0.0    0.0   100.0  100.0
CAF        GHC.IO.Handle.FD        128           0    0.0    0.0     0.0    0.0
CAF        GHC.IO.Encoding.Iconv   120           0    0.0    0.0     0.0    0.0
CAF        GHC.Conc.Signal         110           0    0.0    0.0     0.0    0.0
main      Main                    108           0    0.0    0.0   100.0  100.0
fib      Main                    204     2692537  100.0  100.0   100.0  100.0
```
Cost Centers

Cost centers are annotations on a Haskell program which can be added automatically by the GHC compiler -- using -fprof-auto or by a programmer using {-# SCC "name" #-} <expression>, where "name" is any name you wish and <expression> is any valid Haskell expression:

```haskell
-- Main.hs
main :: IO ()
main = do let l = [1..9999999]
          print $ {-# SCC "print_list" #-} (length l)
```

Compiling with -fprof and running with +RTS -p e.g. ghc -prof -rtsopts Main.hs & & ./Main.hs +RTS -p would produce Main.prof once the program's exited.

Read Optimization online: https://riptutorial.com/haskell/topic/4342/optimization
Chapter 44: Overloaded Literals

Remarks

Integer Literals

is a numeral **without** a decimal point

for example 0, 1, 42, ...

is implicitly applied to `fromInteger` which is part of the `Num` type class so it indeed has type `Num a => a` - that is it can have any type that is an instance of `Num`

Fractional Literals

is a numeral **with** a decimal point

for example 0.0, -0.1111, ...

is implicitly applied to `fromRational` which is part of the `Fractional` type class so it indeed has type `a => a` - that is it can have any type that is an instance of `Fractional`

String Literals

If you add the language extension `OverloadedStrings` to `GHC` you can have the same for `String` literals which then are applied to `fromString` from the `Data.String.IsString` type class

This is often used to replace `String` with `Text` or `ByteString`.

List Literals

Lists can defined with the `[1, 2, 3]` literal syntax. In GHC 7.8 and beyond, this can also be used to define other list-like structures with the `OverloadedLists` extension.

By default, the type of `[]` is:

```
> :t []
[] :: [t]
```

With `OverloadedLists`, this becomes:

```
[] :: GHC.Exts.IsList l => l
```
Examples

Integer Numeral

The type of the literal

Prelude> :t 1
1 :: Num a => a

choosing a concrete type with annotations

You can specify the type as long as the target type is `Num with an annotation:

Prelude> 1 :: Int
1
it :: Int
Prelude> 1 :: Double
1.0
it :: Double
Prelude> 1 :: Word
1
it :: Word

if not the compiler will complain

Prelude> 1 :: String

<interactive>:
  No instance for (Num String) arising from the literal `1'
  In the expression: 1 :: String
  In an equation for `it': it = 1 :: String

Floating Numeral

The type of the literal

Prelude> :t 1.0
1.0 :: Fractional a => a

Choosing a concrete type with annotations

You can specify the type with a `type annotation'. The only requirement is that the type must have a `Fractional instance.

Prelude> 1.0 :: Double
1.0
it :: Double

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Strings

The type of the literal

Without any extensions, the type of a string literal – i.e., something between double quotes – is just a string, aka list of characters:

```
Prelude> :t "foo"
"foo" :: [Char]
```

However, when the `OverloadedStrings` extension is enabled, string literals become polymorphic, similar to number literals:

```
Prelude> :set -XOverloadedStrings
Prelude> :t "foo"
"foo" :: Data.String.IsString t => t
```

This allows us to define values of string-like types without the need for any explicit conversions. In essence, the `OverloadedStrings` extension just wraps every string literal in the generic `fromString` conversion function, so if the context demands e.g. the more efficient `Text` instead of `String`, you don't need to worry about that yourself.

Using string literals

```
{-# LANGUAGE OverloadedStrings #-}

import Data.Text (Text, pack)
import Data.ByteString (ByteString, pack)

withString :: String
withString = "Hello String"

-- The following two examples are only allowed with OverloadedStrings

withText :: Text
withText = "Hello Text"                 -- instead of: withText = Data.Text.pack "Hello Text"

withBS :: ByteString
```

https://riptutorial.com/
withBS = "Hello ByteString"  -- instead of: withBS = Data.ByteString.pack "Hello ByteString"

Notice how we were able to construct values of `Text` and `ByteString` in the same way we construct ordinary `String` (or `[Char]`) Values, rather than using each types `pack` function to encode the string explicitly.

For more information on the `OverloadedStrings` language extension, see the extension documentation.

List Literals

GHC’s `OverloadedLists` extension allows you to construct list-like data structures with the list literal syntax.

This allows you to `Data.Map` like this:

```
> :set -XOverloadedLists
> import qualified Data.Map as M
> M.lookup "foo" {"foo", 1}, {"bar", 2])
Just 1
```

Instead of this (note the use of the extra `M.fromList`):

```
> import Data.Map as M
> M.lookup "foo" (M.fromList {"foo", 1}, {"bar", 2})
Just 1
```

Read Overloaded Literals online: https://riptutorial.com/haskell/topic/369/overloaded-literals
Chapter 45: Parallelism

Parameters

<table>
<thead>
<tr>
<th>Type/Function</th>
<th>Detail</th>
</tr>
</thead>
<tbody>
<tr>
<td>data Eval a</td>
<td>Eval is a Monad that makes it easier to define parallel strategies</td>
</tr>
<tr>
<td>type Strategy a -&gt; a</td>
<td>a function that embodies a parallel evaluation strategy. The function</td>
</tr>
<tr>
<td></td>
<td>traverses (parts of) its argument, evaluating subexpressions in parallel</td>
</tr>
<tr>
<td></td>
<td>or in sequence</td>
</tr>
<tr>
<td>rpar : Strategy a</td>
<td>sparks its argument (for evaluation in parallel)</td>
</tr>
<tr>
<td>rseq : Strategy a</td>
<td>evaluates its argument to weak head normal form</td>
</tr>
<tr>
<td>force : NFData a -&gt; a</td>
<td>evaluates the entire structure of its argument, reducing it to normal</td>
</tr>
<tr>
<td></td>
<td>form, before returning the argument itself. It is provided by the</td>
</tr>
<tr>
<td></td>
<td>Control.DeepSeq module</td>
</tr>
</tbody>
</table>

Remarks

Simon Marlow's book, Concurrent and Parallel Programming in Haskell, is outstanding and covers a multitude of concepts. It is also very much accessible for even the newest Haskell programmer. It is highly recommended and available in PDF or online for free.

Parallel vs Concurrent

Simon Marlow puts it best:

A parallel program is one that uses a multiplicity of computational hardware (e.g., several processor cores) to perform a computation more quickly. The aim is to arrive at the answer earlier, by delegating different parts of the computation to different processors that execute at the same time.

By contrast, concurrency is a program-structuring technique in which there are multiple threads of control. Conceptually, the threads of control execute “at the same time”; that is, the user sees their effects interleaved. Whether they actually execute at the same time or not is an implementation detail; a concurrent program can execute on a single processor through interleaved execution or on multiple physical processors.

Weak Head Normal Form

It’s important to be aware of how lazy-evaluation works. The first section of this chapter will give a strong introduction into WHNF and how this relates to parallel and concurrent programming.
Examples

The Eval Monad

Parallelism in Haskell can be expressed using the Eval Monad from Control.Parallel.Strategies, using the rpar and rseq functions (among others).

```haskell
f1 :: [Int]
f1 = [1..100000000]

f2 :: [Int]
f2 = [1..200000000]

main = runEval $ do
  a <- rpar (f1) -- this'll take a while...
  b <- rpar (f2) -- this'll take a while and then some...
  return (a,b)
```

Running `main` above will execute and "return" immediately, while the two values, `a` and `b` are computed in the background through `rpar`.

Note: ensure you compile with `-threaded` for parallel execution to occur.

rpar

rpar :: Strategy a executes the given strategy (recall: type Strategy a = a -> Eval a) in parallel:

```haskell
import Control.Concurrent
import Control.DeepSeq
import Control.Parallel.Strategies
import Data.List.Ordered

main = loop
  where
    loop = do
      putStrLn "Enter a number"
      n <- getLine
      let lim = read n :: Int
          hf = quot lim 2
          result = runEval $ do
            -- we split the computation in half, so we can concurrently calculate primes
            as <- rpar (force (primesBtwn 2 hf))
            bs <- rpar (force (primesBtwn (hf + 1) lim))
            return (as ++ bs)

            forkIO $ putStrLn ($nPrimes are: " ++ (show result) ++ " for " ++ n ++ "\n")
            loop

      -- Compute primes between two integers
      -- Deliberately inefficient for demonstration purposes
      primesBtwn n m = eratos [n..m]
      where
        eratos [] = []
        eratos (p:xs) = p : eratos (xs `minus` [p, p+p..])
```

https://riptutorial.com/
Running this will demonstrate the concurrent behaviour:

Enter a number
12
Enter a number

Primes are: [2,3,5,7,8,9,10,11,12] for 12

100
Enter a number

Primes are:
[2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100]
for 100

200000000
Enter a number

-- waiting for 200000000
200
Enter a number

Primes are:
for 200

-- still waiting for 200000000

rseq

We can use `rseq :: Strategy a` to force an argument to Weak Head Normal Form:

```haskell
f1 :: [Int]
f1 = [1..100000000]

f2 :: [Int]
f2 = [1..200000000]

main = runEval $ do
  a <- rpar (f1) -- this'll take a while...
  b <- rpar (f2) -- this'll take a while and then some...
  rseq a
  return (a,b)
```

This subtly changes the semantics of the `rpar` example; whereas the latter would return immediately whilst computing the values in the background, this example will wait until `a` can be evaluated to WHNF.

Read Parallelism online: https://riptutorial.com/haskell/topic/6887/parallelism
Chapter 46: Parsing HTML with taggy-lens and lens

Examples

Extract the text contents from a div with a particular id

Taggy-lens allows us to use lenses to parse and inspect HTML documents.

```haskell
#!/usr/bin/env stack
-- stack --resolver lts-7.0 --install-ghc runghc --package text --package lens --package lens

{-# LANGUAGE OverloadedStrings #-}

import qualified Data.Text.Lazy as TL
import qualified Data.Text.IO as T
import Text.Taggy.Lens
import Control.Lens

someHtml :: TL.Text
someHtml = "
<!doctype html><html><body>
<div>first div</div>
<div id="thediv">second div</div>
<div id="not-thediv">third div</div>"

main :: IO ()
main = do
  T.putStrLn (someHtml ^. html . allAttributed (ix "id" . only "thediv") . contents)
```

Filtering elements from the tree

Find div with id="article" and strip out all the inner script tags.

```haskell
#!/usr/bin/env stack
-- stack --resolver lts-7.1 --install-ghc runghc --package text --package lens --package lens

{-# LANGUAGE NoImplicitPrelude #-}
{-# LANGUAGE OverloadedStrings #-}

import ClassyPrelude
import Control.Lens hiding (children, element)
import Data.String.Class (toText, fromText, toString)
import Data.Text (Text)
import Text.Taggy.Lens
import qualified Text.Taggy.Renderer as Renderer

somehtmlSmall :: Text
```
Contribution based upon @duplode’s SO answer

Read Parsing HTML with taggy-lens and lens online:
Chapter 47: Partial Application

Remarks

Let's clear up some misconceptions that beginners might make.

You may have encountered functions such as:

```haskell
max :: (Ord a) => a -> a -> a
max m n
  | m >= n = m
  | otherwise = n
```

Beginners will typically view `max :: (Ord a) => a -> a -> a` as function that takes two arguments (values) of type `a` and returns a value of type `a`. However, what is really happening, is that `max` is taking one argument of type `a` and returning a function of type `a -> a`. This function then takes an argument of type `a` and returns a final value of type `a`.

Indeed, `max` can be written as `max :: (Ord a) => a -> (a -> a)`

Consider the type signature of `max`:

```haskell
Prelude> :t max
max :: Ord a => a -> a -> a

Prelude> :t (max 75)
(max 75) :: (Num a, Ord a) => a -> a

Prelude> :t (max "Fury Road")
(max "Fury Road") :: [Char] -> [Char]

Prelude> :t (max "Fury Road" "Furiosa")
(max "Fury Road" "Furiosa") :: [Char]
```

`max 75` and `max "Fury Road"` may not *look* like functions, but in actuality, they are.

The confusion stems from the fact that in mathematics and many, other, common programming languages, we are allowed to have functions that take multiple arguments. However, in Haskell, functions can only take one argument and they can return either values such as `a`, or functions such as `a -> a`.

Examples

Partially Applied Adding Function

We can use partial application to "lock" the first argument. After applying one argument we are left with a function which expects one more argument before returning the result.
We can then use `addOne` in order to add one to a `Int`.

```haskell
> addOne 5
6
> map addOne [1,2,3]
[2,3,4]
```

## Returning a Partially Applied Function

Returning partially applied functions is one technique to write concise code.

```haskell
add :: Int -> Int -> Int
add x = (+x)
add 5 2
```

In this example `(+x)` is a partially applied function. Notice that the second parameter to the `add` function does not need to be specified in the function definition.

The result of calling `add 5 2` is seven.

## Sections

Sectioning is a concise way to partially apply arguments to infix operators.

For example, if we want to write a function which adds "ing" to the end of a word we can use a section to succinctly define a function.

```haskell
> (++ "ing") "laugh"
"laughing"
```

Notice how we have partially applied the second argument. Normally, we can only partially apply the arguments in the specified order.

We can also use left sectioning to partially apply the first argument.

```haskell
> ("re" ++) "do"
"redo"
```

We could equivalently write this using normal prefix partial application:

```haskell
> ((++) "re") "do"
"redo"
```
A Note on Subtraction

Beginners often incorrectly section negation.

```haskell
> map (-1) [1,2,3]
***error: Could not deduce...
```

This does not work as \(-1\) is parsed as the literal \(-1\) rather than the sectioned operator \(-\) applied to \(1\). The `subtract` function exists to circumvent this issue.

```haskell
> map (subtract 1) [1,2,3]
[0,1,2]
```

Chapter 48: Phantom types

Examples

Use Case for Phantom Types: Currencies

Phantom types are useful for dealing with data, that has identical representations but isn't logically of the same type.

A good example is dealing with currencies. If you work with currencies you absolutely never want to e.g. add two amounts of different currencies. What would the result currency of $5.32 + 2.94$ be? It's not defined and there is no good reason to do this.

A solution to this could look something like this:

```haskell
{-# LANGUAGE GeneralizedNewtypeDeriving #-}

data USD
data EUR

newtype Amount a = Amount Double
  deriving (Show, Eq, Ord, Num)
```

The `GeneralisedNewtypeDeriving` extension allows us to derive `Num` for the `Amount` type. GHC reuses `Double`'s `Num` instance.

Now if you represent Euro amounts with e.g. `(5.0 :: Amount EUR)` you have solved the problem of keeping double amounts separate at the type level without introducing overhead. Stuff like `(1.13 :: Amount EUR) + (5.30 :: Amount USD)` will result in a type error and require you to deal with currency conversion appropriately.

More comprehensive documentation can be found in the [haskell wiki article](https://riptutorial.com/haskell/topic/5227/phantom-types)

Read Phantom types online: [https://riptutorial.com/haskell/topic/5227/phantom-types](https://riptutorial.com/haskell/topic/5227/phantom-types)
Chapter 49: Pipes

Remarks

As the hackage page describes:

pipes is a clean and powerful stream processing library that lets you build and connect reusable streaming components

Programs implemented through streaming can often be succinct and composable, with simple, short functions allowing you to "slot in or out" features easily with the backing of the Haskell type system.

\[
\text{await :: Monad } m \Rightarrow \text{Consumer' } a \text{ m } a
\]

Pulls a value from upstream, where \( a \) is our input type.

\[
\text{yield :: Monad } m \Rightarrow a \rightarrow \text{Producer' } a \text{ m } ()
\]

Produce a value, where \( a \) is the output type.

It's highly recommended you read through the embedded Pipes.Tutorial package which gives an excellent overview of the core concepts of Pipes and how Producer, Consumer and Effect interact.

Examples

Producers

A Producer is some monadic action that can yield values for downstream consumption:

\[
\text{type Producer } b = \text{Proxy } X () () \text{ b}
\text{yield :: Monad } m \Rightarrow a \rightarrow \text{Producer ' a m } ()
\]

For example:

\[
\text{naturals :: Monad } m \Rightarrow \text{Producer } \text{Int } m ()
\text{naturals } = \text{each } [1..] \quad -- \text{each is a utility function exported by Pipes}
\]

We can of course have Producers that are functions of other values too:

\[
\text{naturalsUntil :: Monad } m \Rightarrow \text{Int } \rightarrow \text{Producer } \text{Int } m ()
\text{naturalsUntil } n = \text{each } [1..n]
\]

Consumers

A Consumer can only await values from upstream.
type Consumer a = Proxy () a () X
await :: Monad m => Consumer a m a

For example:
fancyPrint :: MonadIO m => Consumer String m ()
fancyPrint = forever $ do
numStr <- await
liftIO $ putStrLn ("I received: " ++ numStr)

Pipes
Pipes can both await and yield.
type Pipe a b = Proxy () a () b

This Pipe awaits an Int and converts it to a String:
intToStr :: Monad m => Pipe Int String m ()
intToStr = forever $ await >>= (yield . show)

Running Pipes with runEffect
We use runEffect to run our Pipe:
main :: IO ()
main = do
runEffect $ naturalsUntil 10 >-> intToStr >-> fancyPrint

Note that runEffect requires an Effect, which is a self-contained Proxy with no inputs or outputs:
runEffect :: Monad m => Effect m r -> m r
type Effect = Proxy X () () X

(where X is the empty type, also known as Void).

Connecting Pipes
Use >-> to connect Producers, Consumers and Pipes to compose larger Pipe functions.
printNaturals :: MonadIO m => Effect m ()
printNaturals = naturalsUntil 10 >-> intToStr >-> fancyPrint

Producer, Consumer, Pipe,

and Effect types are all defined in terms of the general Proxy type.
Therefore >-> can be used for a variety of purposes. Types defined by the left argument must
match the type consumed by the right argument:
(>->) :: Monad m => Producer b m r -> Consumer b

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m r -> Effect

m r

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The Proxy monad transformer

pipes’s core data type is the Proxy monad transformer. Pipe, Producer, Consumer and so on are defined in terms of Proxy.

Since Proxy is a monad transformer, definitions of Pipes take the form of monadic scripts which await and yield values, additionally performing effects from the base monad m.

Combining Pipes and Network communication

Pipes supports simple binary communication between a client and a server

In this example:

1. a client connects and sends a FirstMessage
2. the server receives and answers DoSomething 0
3. the client receives and answers DoNothing
4. step 2 and 3 are repeated indefinitely

The command data type exchanged over the network:

```haskell
-- Command.hs
{-# LANGUAGE DeriveGeneric #-}
module Command where
import Data.Binary
import GHC.Generics (Generic)
data Command = FirstMessage
  | DoNothing
  | DoSomething Int
deriving (Show, Generic)
instance Binary Command
```

Here, the server waits for a client to connect:

```haskell
module Server where
import Pipes
import qualified Pipes.Binary as PipesBinary
import qualified Pipes.Network.TCP as PNT
import qualified Command as C
import qualified Pipes.Parse as PP
import qualified Pipes.Prelude as PipesPrelude

pageSize :: Int
pageSize = 4096

-- pure handler, to be used with PipesPrelude.map
pureHandler :: C.Command -> C.Command
```

https://riptutorial.com/
pureHandler c = c  -- answers the same command that we have received

-- impure handler, to be used with PipesPrelude.mapM
sideffectHandler :: MonadIO m -> C.Command -> m C.Command
sideffectHandler c = do
  liftIO $ putStrLn $ "received message = " ++ (show c)
  return $ C.DoSomething 0
-- whatever incoming command 'c' from the client, answer DoSomething 0

main :: IO ()
main = PNT.serve (PNT.Host "127.0.0.1") "23456" $
  \(connectionSocket, remoteAddress) -> do
    putStrLn $ "received message = " ++ (show c)
    return $ C.DoSomething 0
-- whatever incoming command 'c' from the client, answer DoSomething 0

The client connects thus:

module Client where

import Pipes
import qualified Pipes.Binary as PipesBinary
import qualified Pipes.Network.TCP as PNT
import qualified Pipes.Prelude as PipesPrelude
import qualified Pipes.Parse as PP
import qualified Command as C

pageSize :: Int
pageSize = 4096

-- pure handler, to be used with PipesPrelude.amp
pureHandler :: C.Command -> C.Command
pureHandler c = c  -- answer the same command received from the server

-- impure handler, to be used with PipesPrelude.mapM
sideffectHandler :: MonadIO m -> C.Command -> m C.Command
sideffectHandler c = do
  liftIO $ putStrLn $ "received: " ++ (show c)
  return C.DoNothing  -- whatever is received from server, answer DoNothing

main :: IO ()
main = PNT.connect ("127.0.0.1") "23456" $
  \(connectionSocket, remoteAddress) -> do
    putStrLn $ "received message = " ++ (show c)
    return $ C.DoSomething 0
-- whatever incoming command 'c' from the client, answer DoSomething 0

sendFirstMessage s = do
  _ <- runEffect $ do
    let encodedProducer = PipesBinary.encode C.FirstMessage
    encodedProducer >>= PNT.toSocket s
  return ()

Read Pipes online: https://riptutorial.com/haskell/topic/6768/pipes
Chapter 50: Profunctor

Introduction

Profunctor is a typeclass provided by the profunctors package in Data.Profunctor.

See the "Remarks" section for a full explanation.

Syntax

- `dimap :: Profunctor p => (a -> b) -> (c -> d) -> p b c -> p a d`
- `lmap :: Profunctor p => (a -> b) -> p b c -> p a c`
- `rmap :: Profunctor p => (b -> c) -> p a b -> p a c`
- `dimap id id = id`
- `lmap id = id`
- `rmap id = id`
- `dimap f g = lmap f . rmap g`
- `lmap f = dimap f id`
- `rmap f = dimap id f`

Remarks

Profunctors are, as described by the docs on Hackage, "a bifunctor where the first argument is contravariant and the second argument is covariant."

So what does this mean? Well, a bifunctor is like a normal functor, except that it has two parameters instead of one, each with its own `fmap`-like function to map on it.

Being "covariant" means that the second argument to a profunctor is just like a normal functor: its mapping function (rmap) has a type signature of `Profunctor p => (b -> c) -> p a b -> p a c`. It just maps the function on the second argument.

Being "contravariant" makes the first argument a little weirder. Instead of mapping like a normal functor, its mapping function (lmap) has a type signature of `Profunctor p => (a -> b) -> p b c -> p a c`. This seemingly backward mapping makes most sense for inputs to a function: you would run `a -> b` on the input, and then your other function, leaving the new input as `a`.

Note: The naming for the normal, one argument functors is a little misleading: the `Functor` typeclass implements "covariant" functors, while "contravariant" functors are implemented in the `Contravariant` typeclass in Data.Functor.Contravariant, and previously the (misleadingly named) `Cofunctor` typeclass in Data.Cofunctor.

Examples

https://riptutorial.com/
(->) Profunctor

(->) is a simple example of a profunctor: the left argument is the input to a function, and the right argument is the same as the reader functor instance.

```haskell
instance Profunctor (->) where
  lmap f g = g . f
  rmap f g = g . g
```

Read Profunctor online: https://riptutorial.com/haskell/topic/9694/profunctor
Chapter 51: Proxies

Examples

Using Proxy

The `Proxy :: k -> *` type, found in `Data.Proxy`, is used when you need to give the compiler some type information - e.g., to pick a type class instance - which is nonetheless irrelevant at runtime.

```
{-# LANGUAGE PolyKinds #-}

data Proxy a = Proxy
```

Functions which use a `Proxy` typically use `ScopedTypeVariables` to pick a type class instance based on the `a` type.

For example, the classic example of an ambiguous function,

```
showread :: String -> String
showread = show . read
```

which results in a type error because the elaborator doesn’t know which instance of `Show` or `Read` to use, can be resolved using `Proxy`:

```
{-# LANGUAGE ScopedTypeVariables #-}

import Data.Proxy

showread :: forall a. (Show a, Read a) => Proxy a -> String -> String
showread _ = (show :: a -> String) . read
```

When calling a function with `Proxy`, you need to use a type annotation to declare which `a` you meant.

```
ghci> showread (Proxy :: Proxy Int) "3"
"3"
ghci> showread (Proxy :: Proxy Bool) "'m'"  -- attempt to parse a char literal as a Bool
"*** Exception: Prelude.read: no parse"
```

The "polymorphic proxy" idiom

Since `Proxy` contains no runtime information, there is never a need to pattern-match on the `Proxy` constructor. So a common idiom is to abstract over the `Proxy` datatype using a type variable.

```
showread :: forall proxy a. (Show a, Read a) => proxy a -> String -> String
showread _ = (show :: a -> String) . read
```
Now, if you happen to have an \( f \, a \) in scope for some \( f \), you don’t need to write out \( \text{Proxy} :: \text{Proxy} \, a \) when calling \( f \).

```
ghci> let chars = "foo"  -- chars :: [Char]
ghci> showread chars "'a'"
"'a'"
```

**Proxy is like ()**

Since \( \text{Proxy} \) contains no runtime information, you can always write a natural transformation \( f \, a \rightarrow \text{Proxy} \, a \) for any \( f \).

```
proxy :: f \, a \rightarrow \text{Proxy} \, a
proxy _ = Proxy
```

This is just like how any given value can always be erased to \( () \):

```
unit :: a \rightarrow ()
unit _ = ()
```

Technically, \( \text{Proxy} \) is the terminal object in the category of functors, just like \( () \) is the terminal object in the category of values.

**Read Proxies online:** [https://riptutorial.com/haskell/topic/8025/proxies](https://riptutorial.com/haskell/topic/8025/proxies)
**Chapter 52: QuickCheck**

**Examples**

**Declaring a property**

At its simplest, a *property* is a function which returns a `Bool`.

```
prop_reverseDoesNotChangeLength xs = length (reverse xs) == length xs
```

A property declares a high-level invariant of a program. The QuickCheck test runner will evaluate the function with 100 random inputs and check that the result is always `True`.

By convention, functions that are properties have names which start with `prop_`.

**Checking a single property**

The `quickCheck` function tests a property on 100 random inputs.

```
ghci> quickCheck prop_reverseDoesNotChangeLength
+++ OK, passed 100 tests.
```

If a property fails for some input, `quickCheck` prints out a counterexample.

```
prop_reverseIsAlwaysEmpty xs = reverse xs == [] -- plainly not true for all xs
ghci> quickCheck prop_reverseIsAlwaysEmpty
*** Failed! Falsifiable (after 2 tests):
[()]
```

**Checking all the properties in a file**

`quickCheckAll` is a Template Haskell helper which finds all the definitions in the current file whose name begins with `prop_` and tests them.

```
{-# LANGUAGE TemplateHaskell #-}

import Test.QuickCheck (quickCheckAll)
import Data.List (sort)

idempotent :: Eq a => (a -> a) -> a -> Bool
idempotent f x = f (f x) == f x

prop_sortIdempotent = idempotent sort

-- does not begin with prop_, will not be picked up by the test runner
sortDoesNotChangeLength xs = length (sort xs) == length xs
```
Randomly generating data for custom types

The `Arbitrary` class is for types that can be randomly generated by QuickCheck.

The minimal implementation of `Arbitrary` is the `arbitrary` method, which runs in the `Gen` monad to produce a random value.

Here is an instance of `Arbitrary` for the following datatype of non-empty lists.

```haskell
import Test.QuickCheck.Arbitrary (Arbitrary(..))
import Test.QuickCheck.Gen (oneof)
import Control.Applicative ((<$>), (<*>))

data NonEmpty a = End a | Cons a (NonEmpty a)

instance Arbitrary a => Arbitrary (NonEmpty a) where
  arbitrary = oneof 
    [  -- randomly select one of the cases from the list
      End <$> arbitrary,  -- call a's instance of Arbitrary
      Cons <$> arbitrary <*> arbitrary  -- recursively call NonEmpty's instance of Arbitrary
    ]
```

Using implication (==>) to check properties with preconditions

```haskell
prop_evenNumberPlusOneIsOdd :: Integer -> Property
prop_evenNumberPlusOneIsOdd x = even x ==> odd (x + 1)
```

If you want to check that a property holds given that a precondition holds, you can use the `==>` operator. Note that if it’s very unlikely for arbitrary inputs to match the precondition, QuickCheck can give up early.

```haskell
prop_overlySpecific x y = x == 0 ==> x * y == 0
ghci> quickCheck prop_overlySpecific
*** Gave up! Passed only 31 tests.
```

Limiting the size of test data

It can be difficult to test functions with poor asymptotic complexity using quickcheck as the random inputs are not usually size bounded. By adding an upper bound on the size of the input we can still
test these expensive functions.

```haskell
import Data.List(permutations)
import Test.QuickCheck

longRunningFunction :: [a] -> Int
longRunningFunction xs = length (permutations xs)

factorial :: Integral a => a -> a
factorial n = product [1..n]

prop_numberOfPermutations xs =
    longRunningFunction xs == factorial (length xs)

ghci> quickCheckWith (stdArgs { maxSize = 10}) prop_numberOfPermutations
```

By using `quickCheckWith` with a modified version of `stdArgs` we can limit the size of the inputs to be at most 10. In this case, as we are generating lists, this means we generate lists of up to size 10. Our permutations function doesn't take too long to run for these short lists but we can still be reasonably confident that our definition is correct.

Read QuickCheck online: https://riptutorial.com/haskell/topic/1156/quickcheck
Chapter 53: Reactive-banana

Examples

Injecting external events into the library

This example is not tied to any concrete GUI toolkit, like reactive-banana-wx does, for instance. Instead it shows how to inject arbitrary IO actions into FRP machinery.

The Control.Event.Handler module provides an `addHandler` function which creates a pair of `AddHandler a` and `a -> IO ()` values. The former is used by reactive-banana itself to obtain an `Event a` value, while the latter is a plain function that is used to trigger the corresponding event.

```haskell
import Data.Char (toUpper)
import Control.Event.Handler
import Reactive.Banana

main = do
  (inputHandler, inputFire) <- newAddHandler

In our case the `a` parameter of the handler is of type `String`, but the code that lets compiler infer that will be written later.

Now we define the EventNetwork that describes our FRP-driven system. This is done using `compile` function:

```haskell
main = do
  (inputHandler, inputFire) <- newAddHandler
  compile $ do
    inputEvent <- fromAddHandler inputHandler
```

The `fromAddHandler` function transforms `AddHandler a` value into a `Event a`, which is covered in the next example.

Finally, we launch our "event loop", that would fire events on user input:

```haskell
main = do
  (inputHandler, inputFire) <- newAddHandler
  compile $ do
    forever $ do
      input <- getLine
      inputFire input
```

Event type

In reactive-banana the `Event` type represents a stream of some events in time. An `Event` is similar to an analog impulse signal in the sense that it is not continuous in time. As a result, `Event` is an
instance of the \texttt{Functor} typeclass only. You can’t combine two \texttt{Event}s together because they may fire at different times. You can do something with an \texttt{Event}’s [current] value and react to it with some \texttt{IO} action.

Transformations on \texttt{Event}s value are done using \texttt{fmap}:

```haskell
main = do
  (inputHandler, inputFire) <- newAddHandler
  compile $ do
    inputEvent <- fromAddHandler inputHandler
    -- turn all characters in the signal to upper case
    let inputEvent' = fmap (map toUpper) inputEvent
```

Reacting to an \texttt{Event} is done the same way. First you \texttt{fmap} it with an action of type \texttt{a -> IO ()} and then pass it to \texttt{reactimate} function:

```haskell
main = do
  (inputHandler, inputFire) <- newAddHandler
  compile $ do
    inputEvent <- fromAddHandler inputHandler
    -- turn all characters in the signal to upper case
    let inputEvent' = fmap (map toUpper) inputEvent
    let inputEventReaction = fmap putStrLn inputEvent' -- this has type `Event (IO ())
    reactimate inputEventReaction
```

Now whenever \texttt{inputFire "something"} is called, "SOMETHING" would be printed.

**Behavior type**

To represent continuous signals, reactive-banana features \texttt{Behavior a} type. Unlike \texttt{Event}, a \texttt{Behavior} is an \texttt{Applicative}, which lets you combine \texttt{n} \texttt{Behavior}s using an \texttt{n}-ary pure function (using \texttt{<*>}).

To obtain a \texttt{Behavior a} from the \texttt{Event a} there is \texttt{accumE} function:

```haskell
main = do
  (inputHandler, inputFire) <- newAddHandler
  compile $ do
    ...
    inputBehavior <- accumE "" $ fmap (\oldValue newValue -> newValue) inputEvent
```

\texttt{accumE} takes \texttt{Behavior}'s initial value and an \texttt{Event}, containing a function that would set it to the new value.

As with \texttt{Event}s, you can use \texttt{fmap} to work with current \texttt{Behavior}s value, but you can also combine them with \texttt{(<*>)}.

```haskell
main = do
  (inputHandler, inputFire) <- newAddHandler
  compile $ do
    ...
    inputBehavior <- accumE "" $ fmap (\oldValue newValue -> newValue) inputEvent
```
inputBehavior' <- accumE "" $ fmap (\oldValue newValue -> newValue) inputEvent
let constantTrueBehavior = (==) <$> inputBehavior <*> inputBehavior'

To react on Behavior changes there is a changes function:

main = do
    (inputHandler, inputFire) <- newAddHandler
    compile $ do ...
    inputBehavior <- accumE "" $ fmap (\oldValue newValue -> newValue) inputEvent
    inputBehavior' <- accumE "" $ fmap (\oldValue newValue -> newValue) inputEvent
    let constantTrueBehavior = (==) <$> inputBehavior <*> inputBehavior'
    inputChanged <- changes inputBehavior

The only thing that should be noted is that changes return Event (Future a) instead of Event a. Because of this, reactimate' should be used instead of reactimate. The rationale behind this can be obtained from the documentation.

Actuating EventNetworks

EventNetworks returned by compile must be actuated before reactivated events have an effect.

main = do
    (inputHandler, inputFire) <- newAddHandler

    eventNetwork <- compile $ do
        inputEvent <- fromAddHandler inputHandler
        let inputEventReaction = fmap putStrLn inputEvent
        reactimate inputEventReaction

        inputFire "This will NOT be printed to the console!"
        actuate eventNetwork
        inputFire "This WILL be printed to the console!"

Read Reactive-banana online: https://riptutorial.com/haskell/topic/4186/reactive-banana
Chapter 54: Reader / ReaderT

Introduction

Reader provides functionality to pass a value along to each function. A helpful guide with some diagrams can be found here: http://adit.io/posts/2013-06-10-three-useful-monads.html

Examples

Simple demonstration

A key part of the Reader monad is the `ask` function, which is defined for illustrative purposes:

```haskell
import Control.Monad.Trans.Reader hiding (ask)
import Control.Monad.Trans

ask :: Monad m => ReaderT r m r
ask = reader id

main :: IO ()
main = do
    let f = (runReaderT $ readerExample) :: Integer -> IO String
    x <- f 100
    print x
    --
    let fIO = (runReaderT $ readerExampleIO) :: Integer -> IO String
    y <- fIO 200
    print y

readerExample :: ReaderT Integer IO String
readerExample = do
    x <- ask
    return $ "The value is: " ++ show x

readerExampleIO :: ReaderT Integer IO String
readerExampleIO = do
    x <- reader id
    lift $ print "Hello from within"
    liftAnnotated $ print "Hello from within..."
    return $ "The value is: " ++ show x
```

The above will print out:

"The value is: 100"
"Hello from within"
"Hello from within..."
"The value is: 200"
Read Reader / ReaderT online: https://riptutorial.com/haskell/topic/9320/reader---readert
Chapter 55: Record Syntax

Examples

Basic Syntax

Records are an extension of sum algebraic data type that allow fields to be named:

```haskell
data StandardType = StandardType String Int Bool --standard way to create a sum type

data RecordType = RecordType { -- the same sum type with record syntax
  aString :: String
, aNumber :: Int
, isTrue  :: Bool
}
```

The field names can then be used to get the named field out of the record

```haskell
> let r = RecordType {aString = "Foobar", aNumber= 42, isTrue = True}
> :t r
  r :: RecordType
> :t aString
  aString :: RecordType -> String
> aString r
  "Foobar"
```

Records can be pattern matched against

```haskell
case r of
  RecordType{aNumber = x, aString=str} -> ...
```

Notice that not all fields need be named

Records are created by naming their fields, but can also be created as ordinary sum types (often useful when the number of fields is small and not likely to change)

```haskell
r  = RecordType {aString = "Foobar", aNumber= 42, isTrue = True}
r' = RecordType  "Foobar" 42 True
```

If a record is created without a named field, the compiler will issue a warning, and the resulting value will be undefined.

```haskell
> let r = RecordType {aString = "Foobar", aNumber= 42}
<interactive>:1:9: Warning:
  Fields of RecordType not initialized: isTrue
> isTrue r
  Error 'undefined'
```

A field of a record can be updated by setting its value. Unmentioned fields do not change.
> let r = RecordType {aString = "Foobar", aNumber= 42, isTrue = True}
> let r' = r{aNumber=117}
-- r'{aString = "Foobar", aNumber= 117, isTrue = True}

It is often useful to create lenses for complicated record types.

### Copying Records while Changing Field Values

Suppose you have this type:

```haskell
data Person = Person { name :: String, age:: Int } deriving (Show, Eq)
```

and two values:

```haskell
alex = Person { name = "Alex", age = 21 }
jenny = Person { name = "Jenny", age = 36 }
```

a new value of type `Person` can be created by copying from `alex`, specifying which values to change:

```haskell
anotherAlex = alex { age = 31 }
```

The values of `alex` and `anotherAlex` will now be:

```haskell
Person {name = "Alex", age = 21}
Person {name = "Alex", age = 31}
```

### Records with newtype

Record syntax can be used with `newtype` with the restriction that there is exactly one constructor with exactly one field. The benefit here is the automatic creation of a function to unwrap the newtype. These fields are often named starting with `run` for monads, `get` for monoids, and `un` for other types.

```haskell
newtype State s a = State { runState :: s -> (s, a) }
newtype Product a = Product { getProduct :: a }
newtype Fancy = Fancy { unfancy :: String }
-- a fancy string that wants to avoid concatenation with ordinary strings
```

It is important to note that the record syntax is typically never used to form values and the field name is used strictly for unwrapping.

```haskell
getProduct $ mconcat [Product 7, Product 9, Product 12]
-- > 756
```
RecordWildCards

{-# LANGUAGE RecordWildCards #-}

data Client = Client { firstName :: String,
                  , lastName  :: String
                  , clientID  :: String
                  } deriving (Show)

printClientName :: Client -> IO ()
printClientName Client{..} = do
    putStrLn firstName
    putStrLn lastName
    putStrLn clientID

The pattern `Client{..}` brings in scope all the fields of the constructor `Client`, and is equivalent to the pattern `Client{ firstName = firstName, lastName = lastName, clientID = clientID }`

It can also be combined with other field matchers like so:

`Client { firstName = "Joe", .. }`

This is equivalent to `Client{ firstName = "Joe", lastName = lastName, clientID = clientID }`

Defining a data type with field labels

It is possible to define a data type with field labels.

data Person = Person { age :: Int, name :: String }

This definition differs from a normal record definition as it also defines `*record accessors*` which can be used to access parts of a data type.

In this example, two record accessors are defined, `age` and `name`, which allow us to access the `age` and `name` fields respectively.

`age :: Person -> Int`  
`name :: Person -> String`

Record accessors are just Haskell functions which are automatically generated by the compiler. As such, they are used like ordinary Haskell functions.

By naming fields, we can also use the field labels in a number of other contexts in order to make our code more readable.
Pattern Matching

```
lowerCaseName :: Person -> String
lowerCaseName (Person { name = x }) = map toLower x
```

We can bind the value located at the position of the relevant field label whilst pattern matching to a new value (in this case `x`) which can be used on the RHS of a definition.

Pattern Matching with `NamedFieldPuns`

```
lowerCaseName :: Person -> String
lowerCaseName (Person { name }) = map toLower name
```

The `NamedFieldPuns` extension instead allows us to just specify the field label we want to match upon, this name is then shadowed on the RHS of a definition so referring to `name` refers to the value rather than the record accessor.

Pattern Matching with `RecordWildcards`

```
lowerCaseName :: Person -> String
lowerCaseName (Person { .. }) = map toLower name
```

When matching using `RecordWildCards`, all field labels are brought into scope. (In this specific example, `name` and `age`)

This extension is slightly controversial as it is not clear how values are brought into scope if you are not sure of the definition of `Person`.

Record Updates

```
setName :: String -> Person -> Person
setName newName person = person { name = newName }
```

There is also special syntax for updating data types with field labels.

Read Record Syntax online: https://riptutorial.com/haskell/topic/1950/record-syntax
Chapter 56: Recursion Schemes

Remarks

Functions mentioned here in examples are defined with varying degrees of abstraction in several packages, for example, data-fix and recursion-schemes (more functions here). You can view a more complete list by searching on Hayoo.

Examples

Fixed points

Fix takes a "template" type and ties the recursive knot, layering the template like a lasagne.

newtype Fix f = Fix { unFix :: f (Fix f) }

Inside a Fix f we find a layer of the template f. To fill in f’s parameter, Fix f plugs in itself. So when you look inside the template f you find a recursive occurrence of Fix f.

Here is how a typical recursive datatype can be translated into our framework of templates and fixed points. We remove recursive occurrences of the type and mark their positions using the r parameter.

{-# LANGUAGE DeriveFunctor #-}

-- natural numbers
-- data Nat = Zero | Suc Nat
data NatF r = Zero_ | Suc_ r deriving Functor
type Nat = Fix NatF

zero :: Nat
zero = Fix Zero_

suc :: Nat -> Nat
suc n = Fix (Suc_ n)

-- lists: note the additional type parameter a
-- data List a = Nil | Cons a (List a)
data ListF a r = Nil_ | Cons_ a r deriving Functor
type List a = Fix (ListF a)

nil :: List a
nil = Fix Nil_

cons :: a -> List a -> List a
cons x xs = Fix (Cons_ x xs)

-- binary trees: note two recursive occurrences
-- data Tree a = Leaf | Node (Tree a) a (Tree a)
data TreeF a r = Leaf_ | Node_ r a r deriving Functor
type Tree a = Fix (TreeF a)
leaf :: Tree a
leaf = Fix Leaf_
node :: Tree a -> a -> Tree a -> Tree a
node l x r = Fix (Node_ l x r)

Folding up a structure one layer at a time

Catamorphisms, or folds, model primitive recursion. cata tears down a fixpoint layer by layer, using an algebra function (or folding function) to process each layer. cata requires a Functor instance for the template type f.

cata :: Functor f => (f a -> a) -> Fix f -> a
cata f = f . fmap (cata f) . unFix

-- list example
foldr :: (a -> b -> b) -> b -> List a -> b
foldr f z = cata alg
  where alg Nil_ = z
        alg (Cons_ x acc) = f x acc

Unfolding a structure one layer at a time

Anamorphisms, or unfolds, model primitive corecursion. ana builds up a fixpoint layer by layer, using a coalgebra function (or unfolding function) to produce each new layer. ana requires a Functor instance for the template type f.

ana :: Functor f => (a -> f a) -> a -> Fix f
ana f = Fix . fmap (ana f) . f

-- list example
unfoldr :: (b -> Maybe (a, b)) -> b -> List a
unfoldr f = ana coalg
  where coalg x = case f x of
          Nothing -> Nil_
          Just (x, y) -> Cons_ x y

Note that ana and cata are dual. The types and implementations are mirror images of one another.

Unfolding and then folding, fused

It's common to structure a program as building up a data structure and then collapsing it to a single value. This is called a hylomorphism or refold. It's possible to elide the intermediate structure altogether for improved efficiency.

hylo :: Functor f => (a -> f a) -> (f b -> b) -> a -> b
hylo f g = g . fmap (hylo f g) . f -- no mention of Fix!

Derivation:

hylo f g = cata g . ana f

https://riptutorial.com/
Primitive recursion

Paramorphisms model primitive recursion. At each iteration of the fold, the folding function receives the subtree for further processing.

\[
\text{para} :: \text{Functor } f \Rightarrow (f (\text{Fix } f, a) \to a) \to \text{Fix } f \to a
\]
\[
\text{para } f = f \cdot \text{fmap } (\lambda x \to (x, \text{para } f x)) \cdot \text{unFix}
\]

The Prelude’s tails can be modelled as a paramorphism.

\[
\text{tails} :: \text{List } a \to \text{List } (\text{List } a)
\]
\[
\text{tails} = \text{para } \text{alg}
\]
\[
\text{where } \text{alg } \text{Nil} = \text{cons } \text{nil } \text{nil} \quad -- []
\]
\[
\text{alg } \text{Cons } x (\text{x}, \text{xss}) = \text{cons } (\text{cons } x \text{xss}) \text{xss} \quad -- (x:xss):xss
\]

Primitive corecursion

Apomorphisms model primitive corecursion. At each iteration of the unfold, the unfolding function may return either a new seed or a whole subtree.

\[
\text{apo} :: \text{Functor } f \Rightarrow (a \to f (\text{Either } (\text{Fix } f) a)) \to a \to \text{Fix } f
\]
\[
\text{apo } f = \text{Fix } f \cdot \text{fmap } (\text{either } \text{id } (\text{apo } f)) \cdot f
\]

Note that apo and para are dual. The arrows in the type are flipped; the tuple in para is dual to the Either in apo, and the implementations are mirror images of each other.

Read Recursion Schemes online: https://riptutorial.com/haskell/topic/2984/recursion-schemes
Chapter 57: Rewrite rules (GHC)

Examples

Using rewrite rules on overloaded functions

In this question, @Viclib asked about using rewrite rules to exploit typeclass laws to eliminate some overloaded function calls:

Mind the following class:

```haskell
class ListIsomorphic l where
  toList    :: l a -> [a]
  fromList  :: [a] -> l a
```

I also demand that \( \text{toList} \circ \text{fromList} = \text{id} \). How do I write rewrite rules to tell GHC to make that substitution?

This is a somewhat tricky use case for GHC's rewrite rules mechanism, because overloaded functions are rewritten into their specific instance methods by rules that are implicitly created behind the scenes by GHC (so something like \( \text{fromList} :: \text{Seq} \ a \rightarrow [a] \) would be rewritten into \( \text{Seq} \$\text{fromList} \) etc.).

However, by first rewriting \( \text{toList} \) and \( \text{fromList} \) into non-inlined non-typeclass methods, we can protect them from premature rewriting, and preserve them until the rule for the composition can fire:

```haskell
{-# RULES
  "protect toList"   toList = toList';
  "protect fromList" fromList = fromList';
  "fromList/toList"  forall x . fromList' (toList' x) = x; #-}

{-# NOINLINE [0] fromList' #-}
fromList' :: (ListIsomorphic l) => [a] -> l a
fromList' = fromList

{-# NOINLINE [0] toList' #-}
toList' :: (ListIsomorphic l) => l a -> [a]
toList' = toList
```

Read Rewrite rules (GHC) online: https://riptutorial.com/haskell/topic/4914/rewrite-rules--ghc-
Chapter 58: Role

Introduction

The TypeFamilies language extension allows the programmer to define type-level functions. What distinguishes type functions from non-GADT type constructors is that parameters of type functions can be non-parametric whereas parameters of type constructors are always parametric. This distinction is important to the correctness of the GeneralizedNewTypeDeriving extension. To explicate this distinction, roles are introduced in Haskell.

Remarks

See also SafeNewtypeDeriving.

Examples

Nominal Role

Haskell Wiki has an example of a non-parametric parameter of a type function:

```haskell
type family Inspect x
type instance Inspect Age = Int
type instance Inspect Int = Bool
```

Here \( x \) is non-parametric because to determine the outcome of applying `Inspect` to a type argument, the type function must inspect \( x \).

In this case, the role of \( x \) is nominal. We can declare the role explicitly with the RoleAnnotations extension:

```haskell
type role Inspect nominal
```

Representational Role

An example of a parametric parameter of a type function:

```haskell
data List a = Nil | Cons a (List a)

type family DoNotInspect x
type instance DoNotInspect x = List x
```

Here \( x \) is parametric because to determine the outcome of applying `DoNotInspect` to a type argument, the type function do not need to inspect \( x \).

In this case, the role of \( x \) is representational. We can declare the role explicitly with the RoleAnnotations extension.
Phantom Role

A phantom type parameter has a phantom role. Phantom roles cannot be declared explicitly.

Read Role online: https://riptutorial.com/haskell/topic/8753.role
Chapter 59: Sorting Algorithms

Examples

Insertion Sort

```haskell
insert :: Ord a => a -> [a] -> [a]
insert x [] = [x]
insert x (y:ys) | x < y   = x:y:ys
                | otherwise = y:(insert x ys)

isort :: Ord a => [a] -> [a]
isort [] = []
isort (x:xs) = insert x (isort xs)
```

Example use:

```haskell
> isort [5,4,3,2,1]
Result:
[1,2,3,4,5]
```

Merge Sort

Ordered merging of two ordered lists

Preserving the duplicates:

```haskell
merge :: Ord a => [a] -> [a] -> [a]
merge xs [] = xs
merge [] ys = ys
merge (x:xs) (y:ys) | x <= y   = x:merge xs (y:ys)
                    | otherwise = y:merge (x:xs) ys
```

Top-down version:

```haskell
msort :: Ord a => [a] -> [a]
msort [] = []
msort [a] = [a]
msort xs = merge (msort (firstHalf xs)) (msort (secondHalf xs))

firstHalf xs = let { n = length xs } in take (div n 2) xs
secondHalf xs = let { n = length xs } in drop (div n 2) xs
```

It is defined this way for clarity, not for efficiency.

Example use:
> msort [3,1,4,5,2]

Result:

[1,2,3,4,5]

**Bottom-up version:**

```hs
msort [] = []
msort xs = go [[x] | x <- xs]
  where
go [a] = a
go xs = go (pairs xs)
pairs (a:b:t) = merge a b : pairs t
pairs t = t
```

**Quicksort**

```hs
qsort :: (Ord a) => [a] -> [a]
qsort [] = []
qsort (x:xs) = qsort [a | a <- xs, a < x] ++ [x] ++
  qsort [b | b <- xs, b >= x]
```

**Bubble sort**

```hs
bsort :: Ord a => [a] -> [a]
bsort s = case bsort' s of
  t | t == s -> t
  | otherwise -> bsort t
  where bsort' (x:x2:xs) | x > x2 = x2:(bsort' (x:xs))
    | otherwise = x:(bsort' (x2:xs))
bsort' s = s
```

**Permutation Sort**

Also known as **bogosort**.

```hs
import Data.List (permutations)

sorted :: Ord a => [a] -> Bool
sorted (x:y:xs) = x <= y && sorted (y:xs)
sorted _ = True

psort :: Ord a => [a] -> [a]
psort = head . filter sorted . permutations
```

Extremely inefficient (on today's computers).

**Selection sort**

[https://riptutorial.com/](https://riptutorial.com/)
Selection sort selects the minimum element, repeatedly, until the list is empty.

```haskell
import Data.List (minimum, delete)

ssort :: Ord t => [t] -> [t]
ssort [] = []
ssort xs = let x = minimum xs
            in x : ssort (delete x xs)
```

Read Sorting Algorithms online: https://riptutorial.com/haskell/topic/2300/sorting-algorithms
Chapter 60: Stack

Examples

Installing Stack

Mac OSX

Using Homebrew:

```
brew install haskell-stack
```

Creating a simple project

To create a project called **helloworld** run:

```
stack new helloworld simple
```

This will create a directory called **helloworld** with the files necessary for a Stack project.

Structure

File structure

A simple project has the following files included in it:

```
➜  helloworld ls
LICENSE    Setup.hs    helloworld.cabal src          stack.yaml
```

In the folder **src** there is a file named **Main.hs**. This is the "starting point" of the **helloworld** project. By default **Main.hs** contains a simple "Hello, World!" program.

**Main.hs**

```
module Main where

main :: IO ()
main = do
  putStrLn "hello world"
```

Running the program

Make sure you are in the directory **helloworld** and run:

```
https://riptutorial.com/ 207
```
stack build # Compile the program
stack exec helloworld # Run the program
# prints "hello world"

Stackage Packages and changing the LTS (resolver) version

Stackage is a repository for Haskell packages. We can add these packages to a stack project.

Adding lens to a project.

In a stack project, there is a file called stack.yaml. In stack.yaml there is a segment that looks like:

```yaml
resolver: lts-6.8
```

Stackage keeps a list of packages for every revision of lts. In our case we want the list of packages for lts-6.8 To find these packages visit:

```bash
https://www.stackage.org/lts-6.8 # if a different version is used, change 6.8 to the correct resolver number.
```

Looking through the packages, there is a Lens-4.13.

We can now add the language package by modifying the section of helloworld.cabal:

```haskell
build-depends: base >= 4.7 && < 5
```

to:

```haskell
build-depends: base >= 4.7 && 5,
   lens == 4.13
```

Obviously, if we want to change a newer LTS (after it's released), we just change the resolver number, eg.:

```bash
resolver: lts-6.9
```

With the next stack build Stack will use the LTS 6.9 version and hence download some new dependencies.

Build and Run a Stack Project

In this example our project name is "helloworld" which was created with stack new helloworld simple

First we have to build the project with stack build and then we can run it with

```bash
stack exec helloworld-exe
```

https://riptutorial.com/
Stack install

By running the command

```
stack install
```

Stack will copy a executable file to the folder

```
/Users/<yourusername>/.local/bin/
```

Profiling with Stack

Configure profiling for a project via stack. First build the project with the `--profile` flag:

```
stack build --profile
```

GHC flags are not required in the cabal file for this to work (like `--prof`). stack will automatically turn on profiling for both the library and executables in the project. The next time an executable runs in the project, the usual `+RTS` flags can be used:

```
stack exec -- my-bin +RTS -p
```

Viewing dependencies

To find out what packages your project directly depends on, you can simply use this command:

```
stack list-dependencies
```

This way you can find out what version of your dependencies where actually pulled down by stack.

Haskell projects frequently find themselves pulling in a lot of libraries indirectly, and sometimes these external dependencies cause problems that you need to track down. If you find yourself with a rogue external dependency that you'd like to identify, you can grep through the entire dependency graph and identify which of your dependencies is ultimately pulling in the undesired package:

```
stack dot --external | grep template-haskell
```

```
stack dot
```
prints out a dependency graph in text form that can be searched. It can also be viewed:

```
stack dot --external | dot -Tpng -o my-project.png
```

You can also set the depth of the dependency graph if you want:

```
stack dot --external --depth 3 | dot -Tpng -o my-project.png
```
Read Stack online: https://riptutorial.com/haskell/topic/2970/stack
Chapter 61: State Monad

Introduction

State monads are a kind of monad that carry a state that might change during each computation run in the monad. Implementations are usually of the form `State s a` which represents a computation that carries and potentially modifies a state of type `s` and produces a result of type `a`, but the term "state monad" may generally refer to any monad which carries a state. The `mtl` and `transformers` package provide general implementations of state monads.

Remarks

Newcomers to Haskell often shy away from the `State` monad and treat it like a taboo—like the claimed benefit of functional programming is the avoidance of state, so don't you lose that when you use `State`? A more nuanced view is that:

- State can be useful in small, controlled doses;
- The `State` type provides the ability to control the dose very precisely.

The reasons being that if you have `action :: State s a`, this tells you that:

- `action` is special because it depends on a state;
- The state has type `s`, so `action` cannot be influenced by any old value in your program—only an `s` or some value reachable from some `s`;
- `runState :: State s a -> s -> (a, s)` puts a "barrier" around the stateful action, so that its effectfulness cannot be observed from outside that barrier.

So this is a good set of criteria for whether to use `State` in particular scenario. You want to see that your code is minimizing the scope of the state, both by choosing a narrow type for `s` and by putting `runState` as close to "the bottom" as possible, (so that your actions can be influenced by as few thing as possible.

Examples

Numbering the nodes of a tree with a counter

We have a tree data type like this:

```haskell
data Tree a = Tree a [Tree a] deriving Show
```

And we wish to write a function that assigns a number to each node of the tree, from an incrementing counter:

```haskell
tag :: Tree a -> Tree (a, Int)
```
The long way

First we'll do it the long way around, since it illustrates the State monad's low-level mechanics quite nicely.

```hs
import Control.Monad.State

-- Function that numbers the nodes of a 'Tree'.
tag :: Tree a -> Tree (a, Int)
tag tree =  
  -- tagStep is where the action happens. This just gets the ball
  -- rolling, with '0' as the initial counter value.
evalState (tagStep tree) 0

-- This is one monadic "step" of the calculation. It assumes that
-- it has access to the current counter value implicitly.
tagStep :: Tree a -> State Int (Tree (a, Int))
tagStep (Tree a subtrees) = do  
  -- The 'get :: State s s' action accesses the implicit state
  -- parameter of the State monad. Here we bind that value to
  -- the variable 'counter'.
  counter <- get

  -- The 'put :: s -> State s ()' sets the implicit state parameter
  -- of the 'State' monad. The next 'get' that we execute will see
  -- the value of 'counter + 1' (assuming no other puts in between).
  put (counter + 1)

  -- Recurse into the subtrees. 'mapM' is a utility function
  -- for executing a monadic actions (like 'tagStep') on a list of
  -- elements, and producing the list of results. Each execution of
  -- 'tagStep' will be executed with the counter value that resulted
  -- from the previous list element's execution.
  subtrees' <- mapM tagStep subtrees

  return $ Tree (a, counter) subtrees'
```

Refactoring

Split out the counter into a postIncrement action

The bit where we are getting the current counter and then putting counter + 1 can be split off into a postIncrement action, similar to what many C-style languages provide:

```hs
postIncrement :: Enum s => State s s
postIncrement = do  
  result <- get
  modify succ
  return result
```

Split out the tree walk into a higher-order function
The tree walk logic can be split out into its own function, like this:

```haskell
mapTreeM :: Monad m => (a -> m b) -> Tree a -> m (Tree b)
mapTreeM action (Tree a subtrees) = do
  a' <- action a
  subtrees' <- mapM (mapTreeM action) subtrees
  return $ Tree a' subtrees'
```

With this and the `postIncrement` function we can rewrite `tagStep`:

```haskell
postIncrement = do
  counter <- liftM (+1) (get)
  put counter
  return counter
```

```haskell
tagStep :: Tree a -> State Int (Tree (a, Int))
tagStep = mapTreeM step
  where step :: a -> State Int (a, Int)
    step a = do
      counter <- postIncrement
      return (a, counter)
```

Use the **Traversable class**

The `mapTreeM` solution above can be easily rewritten into an instance of the `Traversable` class:

```haskell
instance Traversable Tree where
  traverse action (Tree a subtrees) = Tree <$> action a <*> traverse action subtrees
```

Note that this required us to use `Applicative` (the `<*>` operator) instead of `Monad`.

With that, now we can write `tag` like a pro:

```haskell
tag :: Traversable t => t a -> t (a, Int)
tag init t = evalState (traverse step t) 0
  where step a = do
          tag <- postIncrement
          return (a, tag)
```

Note that this works for any `Traversable` type, not just our `Tree` type!

**Getting rid of the Traversable boilerplate**

GHC has a `DeriveTraversable` extension that eliminates the need for writing the instance above:

```haskell
{-# LANGUAGE DeriveFunctor, DeriveFoldable, DeriveTraversable #-}
data Tree a = Tree a [Tree a]
deriving (Show, Functor, Foldable, Traversable)
```

Read State Monad online: [https://riptutorial.com/haskell/topic/5740/state-monad](https://riptutorial.com/haskell/topic/5740/state-monad)
Chapter 62: Streaming IO

Examples

Streaming IO

`io-streams` is a Stream-based library that focuses on the Stream abstraction but for IO. It exposes two types:

- **InputStream**: a read-only smart handle
- **OutputStream**: a write-only smart handle

We can create a stream with `makeInputStream :: IO (Maybe a) -> IO (InputStream a)`. Reading from a stream is performed using `read :: InputStream a -> IO (Maybe a)`, where `Nothing` denotes an EOF:

```haskell
import Control.Monad (forever)
import qualified System.IO.Streams as S
import System.Random (randomRIO)

main :: IO ()
main = do
  is <- S.makeInputStream $ randomInt  -- create an InputStream
  forever $ printStream =<< S.read is  -- forever read from that stream
  return ()

randomInt :: IO (Maybe Int)
randomInt = do
  r <- randomRIO (1, 100)
  return $ Just r

printStream :: Maybe Int -> IO ()
printStream Nothing  = print "Nada!"
printStream (Just a) = putStrLn $ show a
```

Read Streaming IO online: https://riptutorial.com/haskell/topic/4984/streaming-io
Chapter 63: Strictness

Examples

Bang Patterns

Patterns annotated with a bang (!) are evaluated strictly instead of lazily.

```
foo (!x, y) !z = [x, y, z]
```

In this example, \( x \) and \( z \) will both be evaluated to weak head normal form before returning the list. It's equivalent to:

```
foo (x, y) z = x `seq` z `seq` [x, y, z]
```

Bang patterns are enabled using the Haskell 2010 `BangPatterns` language extension.

Normal forms

This example provides a brief overview - for a more in-depth explanation of normal forms and examples, see this question.

Reduced normal form

The reduced normal form (or just normal form, when the context is clear) of an expression is the result of evaluating all reducible subexpressions in the given expression. Due to the non-strict semantics of Haskell (typically called laziness), a subexpression is not reducible if it is under a binder (i.e. a lambda abstraction - \( \lambda x \to \ldots \)). The normal form of an expression has the property that if it exists, it is unique.

In other words, it does not matter (in terms of denotational semantics) in which order you reduce subexpressions. However, the key to writing performant Haskell programs is often ensuring that the right expression is evaluated at the right time, i.e, the understanding the operational semantics.

An expression whose normal form is itself is said to be in normal form.

Some expressions, e.g. `let x = 1:x in x`, have no normal form, but are still productive. The example expression still has a value, if one admits infinite values, which here is the list \([1,1,\ldots]\). Other expressions, such as `let y = 1+y in y`, have no value, or their value is undefined.

Weak head normal form

https://riptutorial.com/
The RNF corresponds to fully evaluating an expression - likewise, the weak head normal form (WHNF) corresponds to evaluating to the head of the expression. The head of an expression \( e \) is fully evaluated if \( e \) is an application \( \text{Con} \ e_1 \ e_2 \ldots \ e_n \) and \( \text{Con} \) is a constructor; or an abstraction \( \lambda \ x \rightarrow e_1 \); or a partial application \( f \ e_1 \ e_2 \ldots \ e_n \), where partial application means \( f \) takes more than \( n \) arguments (or equivalently, the type of \( e \) is a function type). In any case, the subexpressions \( e_1 \ldots e_n \) can be evaluated or unevaluated for the expression to be in WHNF - they can even be undefined.

The evaluation semantics of Haskell can be described in terms of the WHNF - to evaluate an expression \( e \), first evaluate it to WHNF, then recursively evaluate all of its subexpressions from left to right.

The primitive \( \text{seq} \) function is used to evaluate an expression to WHNF. \( \text{seq} \ x \ y \) is denotationally equal to \( y \) (the value of \( \text{seq} \ x \ y \) is precisely \( y \)); furthermore \( x \) is evaluated to WHNF when \( y \) is evaluated to WHNF. An expression can also be evaluated to WHNF with a bang pattern (enabled by the \(-\text{XBangPatterns}\) extension), whose syntax is as follows:

\[
f!\ x \ y = \ldots
\]

In which \( x \) will be evaluated to WHNF when \( f \) is evaluated, while \( y \) is not (necessarily) evaluated. A bang pattern can also appear in a constructor, e.g.

\[
data \ X = \text{Con} \ A \ !B \ C \ldots \ N
\]

in which case the constructor \( \text{Con} \) is said to be strict in the \( B \) field, which means the \( B \) field is evaluated to WHNF when the constructor is applied to sufficient (here, two) arguments.

Lazy patterns

Lazy, or irrefutable, patterns (denoted with the syntax \(~\text{pat}\) ) are patterns that always match, without even looking at the matched value. This means lazy patterns will match even bottom values. However, subsequent uses of variables bound in sub-patterns of an irrefutable pattern will force the pattern matching to occur, evaluating to bottom unless the match succeeds.

The following function is lazy in its argument:

\[
f1 :: \text{Either} \ e \ \text{Int} \rightarrow \text{Int}
f1 ~(\text{Right} \ 1) = 42
\]

and so we get

\[
\lambda\ f1 ~\text{Right} \ 1 \n42
\lambda\ f1 ~\text{Right} \ 2 \n42
\lambda\ f1 ~\text{Left} ~\text{"foo"} \n42
\lambda\ f1 ~\text{error} ~\text{"oops!"} \n42
\]
The following function is written with a lazy pattern but is in fact using the pattern's variable which forces the match, so will fail for \texttt{Left} arguments:

\begin{verbatim}
f2 :: Either e Int -> Int
f2 ~(Right x) = x + 1
\end{verbatim}

\begin{verbatim}
l> f2 (Right 1)
l2
l> f2 (Right 2)
l3
l> f2 (Right (error "oops!"))
*** Exception: oops!
l> f2 (Left "foo")
*** Exception: lazypat.hs:5:1-21: Irrefutable pattern failed for pattern (Right x)
l> f2 (error "oops!")
*** Exception: oops!
\end{verbatim}

\texttt{let} bindings are lazy, behave as irrefutable patterns:

\begin{verbatim}
act1 :: IO ()
act1 = do
  ss <- readLn
  let [s1, s2] = ss :: [String]
  putStrLn "Done"

act2 :: IO ()
act2 = do
  ss <- readLn
  let [s1, s2] = ss
  putStrLn s1
\end{verbatim}

Here \texttt{act1} works on inputs that parse to any list of strings, whereas in \texttt{act2} the \texttt{putStrLn s1} needs the value of \texttt{s1} which forces the pattern matching for \texttt{[s1, s2]}, so it works only for lists of exactly two strings:

\begin{verbatim}
l> act1
> ["foo"]
Done
l> act2
> ["foo"]
*** readIO: no parse ***
\end{verbatim}

\textbf{Strict fields}

In a \texttt{data} declaration, prefixing a type with a bang (\texttt{!}) makes the field a \texttt{strict field}. When the data constructor is applied, those fields will be evaluated to weak head normal form, so the data in the fields is guaranteed to always be in weak head normal form.

Strict fields can be used in both record and non-record types:
data User = User
    { identifier :: !Int
    , firstName :: !Text
    , lastName :: !Text
    }

data T = MkT !Int !Int

Read Strictness online: https://riptutorial.com/haskell/topic/3798(strictness)
Chapter 64: Syntax in Functions

Examples

Guards

A function can be defined using guards, which can be thought of classifying behaviour according to input.

Take the following function definition:

```haskell
absolute :: Int -> Int  -- definition restricted to Ints for simplicity
absolute n = if (n < 0) then (-n) else n
```

We can rearrange it using guards:

```haskell
absolute :: Int -> Int
absolute n
| n < 0 = -n
| otherwise = n
```

In this context `otherwise` is a meaningful alias for `True`, so it should always be the last guard.

Pattern Matching

Haskell supports pattern matching expressions in both function definition and through `case` statements.

A case statement is much like a switch in other languages, except it supports all of Haskell's types.

Let's start simple:

```haskell
longName :: String -> String
longName name = case name of
    "Alex"  -> "Alexander"
    "Jenny" -> "Jennifer"
    _       -> "Unknown"  -- the "default" case, if you like
```

Or, we could define our function like an equation which would be pattern matching, just without using a `case` statement:

```haskell
longName "Alex"  = "Alexander"
longName "Jenny" = "Jennifer"
longName _       = "Unknown"
```

A more common example is with the `Maybe` type:
data Person = Person { name :: String, petName :: (Maybe String) }

hasPet :: Person -> Bool
hasPet (Person _ Nothing) = False
hasPet _ = True  -- Maybe can only take `Just a` or `Nothing`, so this wildcard suffices

Pattern matching can also be used on lists:

isEmptyList :: [a] -> Bool
isEmptyList [] = True
isEmptyList _  = False

addFirstTwoItems :: [Int] -> [Int]
addFirstTwoItems [] = []
addFirstTwoItems (x:[]) = [x]
addFirstTwoItems (x:y:ys) = (x + y) : ys

Actually, Pattern Matching can be used on any constructor for any type class. E.g. the constructor for lists is : and for tuples ,

Using where and guards

Given this function:

annualSalaryCalc :: (RealFloat a) => a -> a -> String
annualSalaryCalc hourlyRate weekHoursOfWork
  | hourlyRate * (weekHoursOfWork * 52) <= 40000  = "Poor child, try to get another job"
  | hourlyRate * (weekHoursOfWork * 52) <= 120000 = "Money, Money, Money!"
  | hourlyRate * (weekHoursOfWork * 52) <= 200000 = "Ri¢hie Ri¢h"
  | otherwise = "Hello Elon Musk!"

We can use where to avoid the repetition and make our code more readable. See the alternative function below, using where:

annualSalaryCalc' :: (RealFloat a) => a -> a -> String
annualSalaryCalc' hourlyRate weekHoursOfWork
  | annualSalary <= smallSalary  = "Poor child, try to get another job"
  | annualSalary <= mediumSalary = "Money, Money, Money!"
  | annualSalary <= highSalary   = "Ri¢hie Ri¢h"
  | otherwise = "Hello Elon Musk!"
  where
    annualSalary = hourlyRate * (weekHoursOfWork * 52)
    (smallSalary, mediumSalary, highSalary)  = (40000, 120000, 200000)

As observed, we used the where in the end of the function body eliminating the repetition of the calculation (hourlyRate * (weekHoursOfWork * 52)) and we also used where to organize the salary range.

The naming of common sub-expressions can also be achieved with let expressions, but only the where syntax makes it possible for guards to refer to those named sub-expressions.

Read Syntax in Functions online: https://riptutorial.com/haskell/topic/3799/syntax-in-functions
What is Template Haskell?

Template Haskell refers to the template meta-programming facilities built into GHC Haskell. The paper describing the original implementation can be found [here](#).

What are stages? (Or, what is the stage restriction?)

Stages refer to when code is executed. Normally, code is executed only at runtime, but with Template Haskell, code can be executed at compile time. "Normal" code is stage 0 and compile-time code is stage 1.

The stage restriction refers to the fact that a stage 0 program may not be executed at stage 1 - this would be equivalent to being able to run any regular program (not just meta-program) at compile time.

By convention (and for the sake of implementation simplicity), code in the current module is always stage 0 and code imported from all other modules is stage 1. For this reason, only expressions from other modules may be spliced.

Note that a stage 1 program is a stage 0 expression of type `Q Exp`, `Q Type`, etc; but the converse is not true - not every value (stage 0 program) of type `Q Exp` is a stage 1 program.

Furthermore, since splices can be nested, identifiers can have stages greater than 1. The stage restriction can then be generalized - a stage $n$ program may not be executed in any stage $m>n$. For example, one can see references to such stages greater than 1 in certain error messages:

```haskell
:t [| \x -> $x |]
<interactive>:1:10: error:
  * Stage error: `x' is bound at stage 2 but used at stage 1
  * In the untyped splice: $x
    In the Template Haskell quotation [| \ x -> $x |]
```

Using Template Haskell causes not-in-scope errors from unrelated identifiers?
Normally, all the declarations in a single Haskell module can be thought of as all being mutually recursive. In other words, every top-level declaration is in the scope of every other in a single module. When Template Haskell is enabled, the scoping rules change - the module is instead broken into groups of code separated by TH splices, and each group is mutually recursive, and each group in the scope of all further groups.

Examples

The Q type

The \( Q \) :: * \rightarrow * \) type constructor defined in Language.Haskell.TH.Syntax is an abstract type representing computations which have access to the compile-time environment of the module in which the computation is run. The \( Q \) type also handles variable substitution, called name capture by TH (and discussed here.) All splices have type \( Q \times \) for some \( x \).

The compile-time environment includes:

- in-scope identifiers and information about said identifiers,
  - types of functions
  - types and source data types of constructors
  - full specification of type declarations (classes, type families)
- the location in the source code (line, column, module, package) where the splice occurs
- fixities of functions (GHC 7.10)
- enabled GHC extensions (GHC 8.0)

The \( Q \) type also has the ability to generate fresh names, with the function \( \text{newName} :: \text{String} \rightarrow Q \text{Name} \). Note that the name is not bound anywhere implicitly, so the user must bind it themselves, and so making sure the resulting use of the name is well-scoped is the responsibility of the user.

\( Q \) has instances for Functor, Monad, Applicative and this is the main interface for manipulating \( Q \) values, along with the combinators provided in Language.Haskell.TH.Lib, which define a helper function for every constructor of the TH ast of the form:

```haskell
LitE :: Lit \rightarrow Exp
litE :: Lit \rightarrow ExpQ
AppE :: Exp \rightarrow Exp \rightarrow Exp
appE :: ExpQ \rightarrow ExpQ \rightarrow ExpQ
```

Note that \( \text{ExpQ}, \text{TypeQ}, \text{DecsQ} \) and \( \text{PatQ} \) are synonyms for the AST types which are typically stored inside the \( Q \) type.

The TH library provides a function \( \text{runQ} :: \text{Quasi m} \rightarrow Q \text{a} \rightarrow m \text{a} \), and there is an instance \( \text{Quasi IO} \), so it would seem that the \( Q \) type is just a fancy \( IO \). However, the use of \( \text{runQ} :: Q \text{a} \rightarrow IO \text{a} \) produces an \( IO \) action which does not have access to any compile-time environment - this is only available in the actual \( Q \) type. Such \( IO \) actions will fail at runtime if trying to access said environment.

https://riptutorial.com/
An n-arity curry

The familiar

\[
\text{curry} :: ((a, b) \to c) \to a \to b \to c
\]
\[
\text{curry} = \\lambda a \ b \to f \ (a, b)
\]

function can be generalized to tuples of arbitrary arity, for example:

\[
\text{curry3} :: ((a, b, c) \to d) \to a \to b \to c \to d
\]
\[
\text{curry4} :: ((a, b, c, d) \to e) \to a \to b \to c \to d \to e
\]

However, writing such functions for tuples of arity 2 to (e.g.) 20 by hand would be tedious (and
ignoring the fact that the presence of 20 tuples in your program almost certainly signal design
issues which should be fixed with records).

We can use Template Haskell to produce such \text{curryN} functions for arbitrary \(n\):

{*-# LANGUAGE TemplateHaskell #-*}
import Control.Monad (replicateM)
import Language.Haskell.TH (ExpQ, newName, Exp(..), Pat(..))
import Numeric.Natural (Natural)

\[
\text{curryN} :: \text{Natural} \to Q \text{ Exp}
\]

The \text{curryN} function takes a natural number, and produces the curry function of that arity, as a
Haskell AST.

\[
\text{curryN} \ n \ = \ do
f \ <- \ newName \ "f"
xs \ <- \ replicateM \ (fromIntegral \ n) \ (newName \ "x")
\]

First we produces fresh type variables for each of the arguments of the function - one for the input
function, and one for each of the arguments to said function.

\[
\text{let args = map VarP (f:xs)}
\]

The expression \text{args} represents the pattern \(f \ x1 \ x2 \ldots \ xn\). Note that a pattern is separate syntactic
entity - we could take this same pattern and place it in a lambda, or a function binding, or even the
LHS of a let binding (which would be an error).

\[
\text{ntup} = \text{TupE (map VarE xs)}
\]

The function must build the argument tuple from the sequence of arguments, which is what we’ve
done here. Note the distinction between pattern variables (VarP) and expression variables (VarE).

\[
\text{return } \ $ \text{LamE args (AppE (VarE f) ntup)}
\]

Finally, the value which we produce is the AST \(\lambda x1 \ x2 \ldots x_n \to f \ (x1, x2, \ldots, x_n)\).
We could have also written this function using quotations and 'lifted' constructors:

```haskell
import Language.Haskell.TH.Lib

curryN' :: Natural -> ExpQ
curryN' n = do
  f <- newName "f"
  xs <- replicateM (fromIntegral n) (newName "x")
  lamE (map varP (f:xs))
  [| $(varE f) $(tupE (map varE xs)) |]
```

Note that quotations must be syntactically valid, so `(| \ $(map varP (f:xs)) -- > .. |)` is invalid, because there is no way in regular Haskell to declare a 'list' of patterns - the above is interpreted as `\ var --> ..` and the spliced expression is expected to have type `PatQ`, i.e. a single pattern, not a list of patterns.

Finally, we can load this TH function in GHCi:

```haskell
> :set -XTemplateHaskell
> :t $(curryN 5)
$(curryN 5)
  :: ((t1, t2, t3, t4, t5) -> t) -> t1 -> t2 -> t3 -> t4 -> t5 -> t

> $(curryN 5) \((a,b,c,d,e) --> a+b+c+d+e\) 1 2 3 4 5
15
```

This example is adapted primarily from [here](https://riptutorial.com/).

**Syntax of Template Haskell and Quasiquotes**

Template Haskell is enabled by the `-XTemplateHaskell` GHC extension. This extension enables all the syntactic features further detailed in this section. The full details on Template Haskell are given by the user guide.

---

**Splices**

- A splice is a new syntactic entity enabled by Template Haskell, written as `$(...)`, where `(...)` is some expression.

- There must not be a space between `$` and the first character of the expression; and Template Haskell overrides the parsing of the `$` operator - e.g. `$g f$` is normally parsed as `($)` whereas with Template Haskell enabled, it is parsed as a splice.

- When a splice appears at the top level, the `$` may be omitted. In this case, the spliced expression is the entire line.

- A splice represents code which is run at compile time to produce a Haskell AST, and that AST is compiled as Haskell code and inserted into the program.
• Splices can appear in place of: expressions, patterns, types, and top-level declarations. The type of the spliced expression, in each case respectively, is \( Q \text{Exp} \), \( Q \text{Pat} \), \( Q \text{Type} \), \( Q \text{[Decl]} \). Note that declaration splices may *only* appear at the top level, whereas the others may be inside other expressions, patterns, or types, respectively.

**Expression quotations (note: *not* a QuasiQuotation)**

• An expression quotation is a new syntactic entity written as one of:
  
  - \([e|..|] - ..\) is an expression and the quotation has type \( Q \text{Exp} \);
  - \([p|..|] - ..\) is a pattern and the quotation has type \( Q \text{Pat} \);
  - \([t|..|] - ..\) is a type and the quotation has type \( Q \text{Type} \);
  - \([d|..|] - ..\) is a list of declarations and the quotation has type \( Q \text{[Decl]} \).

• An expression quotation takes a compile time program and produces the AST represented by that program.

• The use of a value in a quotation (e.g. \(\text{x} \rightarrow [| \text{x} |]\)) without a splice corresponds to syntactic sugar for \(\text{x} \rightarrow [| \text{lift}(\text{x}) |]\), where \(\text{lift} :: \text{Lift t} \rightarrow \text{t} \rightarrow Q \text{Exp}\).

```haskell
class Lift t where
    lift :: t -> Q Exp
default lift :: Data t -> t -> Q Exp
```

**Typed splices and quotations**

• Typed splices are similar to previously mentioned (untyped) splices, and are written as \(\$$\{..\}\) where \(\{..\}\) is an expression.

• If \(e\) has type \(Q \text{(TExp a)}\) then \(\$$e\) has type \(a\).

• Typed quotations take the form \(\[[\{\}..\}\]\) where \(\{..\}\) is an expression of type \(a\); the resulting quotation has type \(Q \text{(TExp a)}\).

• Typed expression can be converted to untyped ones: \(\text{unType} :: \text{TExp a} \rightarrow \text{Exp}\).

**QuasiQuotes**

• QuasiQuotes generalize expression quotations - previously, the parser used by the expression quotation is one of a fixed set \((e, p, t, d)\), but QuasiQuotes allow a custom parser to be defined and used to produce code at compile time. Quasi-quotations can appear in all the same contexts as regular quotations.
A quasi-quotiation is written as \([iden|...|]\), where \(iden\) is an identifier of type
\(\text{Language.Haskell.TH.Quote.QuasiQuoter}\).

A \textit{QuasiQuoter} is simply composed of four parsers, one for each of the different contexts in which quotations can appear:

```haskell
data QuasiQuoter = QuasiQuoter { quoteExp :: String -> Q Exp,
quotePat :: String -> Q Pat,
quoteType :: String -> Q Type,
quoteDec :: String -> Q [Dec] }
```

### Names

Haskell identifiers are represented by the type \(\text{Language.Haskell.TH.Syntax.Name}\). Names form the leaves of abstract syntax trees representing Haskell programs in Template Haskell.

An identifier which is currently in scope may be turned into a name with either: \('e\) or \(\textbackslash'T\). In the first case, \(e\) is interpreted in the expression scope, while in the second case \(T\) is in the type scope (recalling that types and value constructors may share the name without ambiguity in Haskell).

Read Template Haskell & QuasiQuotes online: https://riptutorial.com/haskell/topic/5216/template-haskell---quasiquotes
Chapter 66: Testing with Tasty

Examples

SmallCheck, QuickCheck and HUnit

```haskell
import Test.Tasty
import Test.Tasty.SmallCheck as SC
import Test.Tasty.QuickCheck as QC
import Test.Tasty.HUnit

main :: IO ()
main = defaultMain tests

tests :: TestTree
tests = testGroup "Tests" [smallCheckTests, quickCheckTests, unitTests]

smallCheckTests :: TestTree
smallCheckTests = testGroup "SmallCheck Tests"
  [ SC.testProperty "String length <= 3" $ \s -> length (take 3 (s :: String)) <= 3
  , SC.testProperty "String length <= 2" $ -- should fail
    \s -> length (take 3 (s :: String)) <= 2
  ]

quickCheckTests :: TestTree
quickCheckTests = testGroup "QuickCheck Tests"
  [ QC.testProperty "String length <= 5" $ \s -> length (take 5 (s :: String)) <= 5
  , QC.testProperty "String length <= 4" $ -- should fail
    \s -> length (take 5 (s :: String)) <= 4
  ]

unitTests :: TestTree
unitTests = testGroup "Unit Tests"
  [ testCase "String comparison 1" $ 
    assertEqual "description" "OK" "OK"
  , testCase "String comparison 2" $ -- should fail
    assertEqual "description" "fail" "fail!"
  ]
```

Install packages:

```bash
cabal install tasty-smallcheck tasty-quickcheck tasty-hunit
```

Run with cabal:

```bash
cabal exec runhaskell test.hs
```

Read Testing with Tasty online: https://riptutorial.com/haskell/topic/3816/testing-with-tasty
Chapter 67: Traversable

Introduction

The Traversable class generalises the function formerly known as `mapM :: Monad m => (a -> m b) -> [a] -> m [b]` to work with Applicative effects over structures other than lists.

Examples

Instantiating Functor and Foldable for a Traversable structure

```haskell
import Data.Traversable as Traversable

data MyType a =  -- ...
instance Traversable MyType where
    traverse = -- ...

Every Traversable structure can be made a Foldable Functor using the `fmapDefault` and `foldMapDefault` functions found in `Data.Traversable`.

```haskell
instance Functor MyType where
    fmap = Traversable.fmapDefault

instance Foldable MyType where
    foldMap = Traversable.foldMapDefault

fmapDefault is defined by running `traverse` in the Identity applicative functor.

```haskell
newtype Identity a = Identity { runIdentity :: a }

instance Applicative Identity where
    pure = Identity
    Identity f <*> Identity x = Identity (f x)

fmapDefault :: Traversable t => (a -> b) -> t a -> t b
fmapDefault f = runIdentity . traverse (Identity . f)

foldMapDefault is defined using the Const applicative functor, which ignores its parameter while accumulating a monoidal value.

```haskell
newtype Const c a = Const { getConst :: c }

instance Monoid m => Applicative (Const m) where
    pure _ = Const mempty
    Const x <> Const y = Const (x `mappend` y)

foldMapDefault :: (Traversable t, Monoid m) => (a -> m) -> t a -> m
foldMapDefault f = getConst . traverse (Const . f)
```
An instance of `Traversable` for a binary tree

Implementations of `traverse` usually look like an implementation of `fmap` lifted into an `Applicative` context.

```haskell
data Tree a = Leaf
  | Node (Tree a) a (Tree a)

instance Traversable Tree where
  traverse f Leaf = pure Leaf
  traverse f (Node l x r) = Node <$> traverse f l <*> f x <*> traverse f r
```

This implementation performs an **in-order traversal** of the tree.

```haskell
ghci> let myTree = Node (Node Leaf 'a' Leaf) 'b' (Node Leaf 'c' Leaf)
   --    +-'b'--+
   --    |       |
   -- +-'a'-+ +-'c'-+
   -- |     | |     |
   -- *     * *     *
ghci> traverse print myTree
 'a'
 'b'
 'c'
```

The `DeriveTraversable` extension allows GHC to generate `Traversable` instances based on the structure of the type. We can vary the order of the machine-written traversal by adjusting the layout of the `Node` constructor.

```haskell
data Inorder a = ILeaf
  | INode (Inorder a) a (Inorder a) -- as before
deriving (Functor, Foldable, Traversable) -- also using DeriveFunctor and DeriveFoldable

data Preorder a = PrLeaf
  | PrNode a (Preorder a) (Preorder a)
deriving (Functor, Foldable, Traversable)

data Postorder a = PoLeaf
  | PoNode (Postorder a) (Postorder a) a
deriving (Functor, Foldable, Traversable)

-- injections from the earlier Tree type
inorder :: Tree a -> Inorder a
inorder Leaf = ILeaf
inorder (Node l x r) = INode (inorder l) x (inorder r)

preorder :: Tree a -> Preorder a
preorder Leaf = PrLeaf
preorder (Node l x r) = PrNode x (preorder l) (preorder r)

postorder :: Tree a -> Postorder a
postorder Leaf = PoLeaf
postorder (Node l x r) = PoNode (postorder l) (postorder r) x
```

https://riptutorial.com/
Traversing a structure in reverse

A traversal can be run in the opposite direction with the help of the Backwards applicative functor, which flips an existing applicative so that composed effects take place in reversed order.

```
newtype Backwards f a = Backwards { forwards :: f a }

instance Applicative f => Applicative (Backwards f) where
  pure = Backwards . pure
  Backwards ff <*> Backwards fx = Backwards ((\x f -> f x) <$> fx <*> ff)
```

Backwards can be put to use in a "reversed traverse". When the underlying applicative of a traverse call is flipped with Backwards, the resulting effect happens in reverse order.

```
newtype Reverse t a = Reverse { getReverse :: t a }

instance Traversable t => Traversable (Reverse t) where
  traverse f = fmap Reverse . forwards . traverse (Backwards . f) . getReverse

ghci> traverse print (Reverse "abc")
'c'
'b'
'a'
```

The Reverse newtype is found under Data.Functor.Reverse.

Definition of Traversable

```
class (Functor t, Foldable t) => Traversable t where
  {-# MINIMAL traverse | sequenceA #-}
  traverse :: Applicative f => (a -> f b) -> t a -> f (t b)
  traverse f = sequenceA . fmap f

  sequenceA :: Applicative f => t (f a) -> f (t a)
  sequenceA = traverse id

  mapM :: Monad m => (a -> m b) -> t a -> m (t b)
  mapM = traverse

  sequence :: Monad m => t (m a) -> m (t a)
```

https://riptutorial.com/
Traversable structures are finitary containers of elements which can be operated on with an effectful "visitor" operation. The visitor function \( f : a \to f b \) performs a side-effect on each element of the structure and traverse composes those side-effects using Applicative. Another way of looking at it is that \( \text{sequenceA} \) says Traversable structures commute with Applicatives.

Transforming a Traversable structure with the aid of an accumulating parameter

The two \text{mapAccum} functions combine the operations of folding and mapping.

\[
\text{mapAccumL, mapAccumR :: Traversable } t \Rightarrow (a \to b \to (a, c)) \to a \to t b \to (a, t c)
\]

These functions generalise \text{fmap} in that they allow the mapped values to depend on what has happened earlier in the fold. They generalise \text{foldl} / \text{foldr} in that they map the structure in place as well as reducing it to a value.

For example, \text{tails} can be implemented using \text{mapAccumR} and its sister \text{inits} can be implemented using \text{mapAccumL}.

\[
\text{tails, inits :: } [a] \to [[[a]]]
\]

\[
\text{tails} = \text{uncurry } (:) \cdot \text{mapAccumR } (\lambda x \to (x:xs, xs)) []
\]
\[
\text{inits} = \text{uncurry } \text{snoc} \cdot \text{mapAccumL } (\lambda x \to (x \cdot \text{snoc } xs, xs)) []
\]

\[
\text{where } \text{snoc } x xs = xs ++ [x]
\]

\[
\text{ghci> tails } "abc"
["abc", "bc", "c", ""
\]
\[
\text{ghci> inits } "abc"
["", "a", "ab", "abc"
\]

\text{mapAccumL} is implemented by traversing in the \text{State} applicative functor.

\[
\{-# LANGUAGE DeriveFunctor #-\}
\]

\[
\text{newtype State } s a = \text{State } \{ \text{runState } : s \to (s, a) \} \text{ deriving Functor}
\]
\[
\text{instance Applicative (State } s) \text{ where}
\]
\[
\text{pure } x = \text{State } \$ \ s \to (s, x)
\]
\[
\text{State } ff \ <\> \text{ State } fx = \text{State } \$ \ s \to \let\ t, f) = ff s
\]
\[
\ (u, x) = fx t
\]
\[
\in (u, f x)
\]
mapAccumL \( f \) \( z \) \( t \) = runState \( \text{traverse} \ (\text{State} \ . \ \text{flip} \ f) \ t \) \( z \)

mapAccumR works by running \( \text{mapAccumL} \) in reverse.

\[
\text{mapAccumR} \ f \ z = \text{fmap} \ \text{getReverse} \ . \ \text{mapAccumL} \ f \ z \ . \ \text{Reverse}
\]

Traversable structures as shapes with contents

If a type \( t \) is \text{Traversable} then values of \( t \ a \) can be split into two pieces: their "shape" and their "contents":

\[
\text{data} \ \text{Traversed} \ t \ a = \text{Traversed} \ \{ \ \text{shape} :: t (), \ \text{contents} :: [a] \ \}\n\]

where the "contents" are the same as what you'd "visit" using a \text{Foldable} instance.

Going one direction, from \( t \ a \) to \text{Traversed} \( t \ a \) doesn't require anything but \text{Functor} and \text{Foldable}

\[
\text{break} :: (\text{Functor} \ t, \ \text{Foldable} \ t) \Rightarrow t \ a \rightarrow \text{Traversed} \ t \ a
\]

\[
\text{break} \ ta = \text{Traversed} \ (\text{fmap} \ (\const ())) \ ta \ (\text{toList} \ ta)
\]

but going back uses the \text{traverse} function crucially

import Control.Monad.State

\[
\text{-- invariant: state is non-empty}
\]

\[
\text{pop} :: \text{State} [a] \ a
\]

\[
\text{pop} = \text{state} \ ((a:as) \rightarrow (a, as))
\]

\[
\text{recombine} :: \text{Traversable} \ t \Rightarrow \text{Traversed} \ t \ a \rightarrow t \ a
\]

\[
\text{recombine} \ (\text{Traversed} \ s \ c) = \text{evalState} \ (\text{traverse} \ (\const \ \text{pop}) \ s) \ c
\]

The \text{Traversable} laws require that \text{break} \ . \ \text{recombine} \ and \ \text{recombine} \ . \ \text{break} \ are both identity. Notably, this means that there are exactly the right number elements in \text{contents} to fill \text{shape} completely with no left-overs.

\text{Traversed} \ t \ is \text{Traversable} itself. The implementation of \text{traverse} works by visiting the elements using the list's instance of \text{Traversable} and then reattaching the inert shape to the result.

\[
\text{instance} \ \text{Traversable} \ (\text{Traversed} \ t) \ \text{where}
\]

\[
\text{traverse} \ f \ (\text{Traversed} \ s \ c) = \text{fmap} \ (\text{Traversed} \ s) \ (\text{traverse} \ f \ c)
\]

Transposing a list of lists

Noting that \text{zip} transposes a tuple of lists into a list of tuples,

\[
\text{ghci} > \text{uncurry} \ \text{zip} \ ([1,2],[3,4])
\]

\[
[(1,3), (2,4)]
\]
and the similarity between the types of `transpose` and `sequenceA`,

```haskell
-- transpose exchanges the inner list with the outer list
-- +---+-->--+-+
-- | | | | |
transpose :: [[a]] -> [[a]]
-- | | | | |
-- +---+-->--+-+

-- sequenceA exchanges the inner Applicative with the outer Traversable
-- +-------+-----+
-- | | | |
sequenceA :: (Traversable t, Applicative f) => t (f a) -> f (t a)
-- | | |
-- +-------+-----+
```

the idea is to use `[]`'s `Traversable` and `Applicative` structure to deploy `sequenceA` as a sort of `n-ary` `zip`, zipping together all the inner lists together pointwise.

`[]`'s default "prioritised choice" `Applicative` instance is not appropriate for our use - we need a "zippy" `Applicative`. For this we use the ZipList newtype, found in `Control.Applicative`.

```haskell
newtype ZipList a = ZipList { getZipList :: [a] }

instance Applicative ZipList where
  pure x = ZipList (repeat x)
  ZipList fs <*> ZipList xs = ZipList (zipWith ($) fs xs)
```

Now we get `transpose` for free, by traversing in the ZipList Applicative.

```haskell
transpose :: [[a]] -> [[a]]
transpose = getZipList . traverse ZipList

ghci> let myMatrix = [[1,2,3],[4,5,6],[7,8,9]]
ghci> transpose myMatrix
[[1,4,7],[2,5,8],[3,6,9]]
```

Read Traversable online: [https://riptutorial.com/haskell/topic/754/traversable](https://riptutorial.com/haskell/topic/754/traversable)
Chapter 68: Tuples (Pairs, Triples, ...)

Remarks

- Haskell does not support tuples with one component natively.
- Units (written ()) can be understood as tuples with zero components.
- There are no predefined functions to extract components of tuples with more than two components. If you feel that you need such functions, consider using a custom data type with record labels instead of the tuple type. Then you can use the record labels as functions to extract the components.

Examples

Construct tuple values

Use parentheses and commas to create tuples. Use one comma to create a pair.

(1, 2)

Use more commas to create tuples with more components.

(1, 2, 3)

(1, 2, 3, 4)

Note that it is also possible to declare tuples using in their unsugared form.

(,) 1 2       -- equivalent to (1,2)
(,,) 1 2 3    -- equivalent to (1,2,3)

Tuples can contain values of different types.

("answer", 42, '?')

Tuples can contain complex values such as lists or more tuples.

([1, 2, 3], "hello", ('A', 65))

(1, (2, (3, 4), 5), 6)

Write tuple types

Use parentheses and commas to write tuple types. Use one comma to write a pair type.
Use more commas to write tuple types with more components.

Tuples can contain values of different types.

Tuples can contain complex values such as lists or more tuples.

Pattern Match on Tuples

Pattern matching on tuples uses the tuple constructors. To match a pair for example, we’d use the `(,)` constructor:

```haskell
myFunction1 (a, b) = ...
```

We use more commas to match tuples with more components:

```haskell
myFunction2 (a, b, c) = ...
myFunction3 (a, b, c, d) = ...
```

Tuple patterns can contain complex patterns such as list patterns or more tuple patterns.

```haskell
myFunction4 ([a, b, c], d, e) = ...
myFunction5 (a, (b, (c, d), e), f) = ...
```

Extract tuple components

Use the `fst` and `snd` functions (from `Prelude` or `Data.Tuple`) to extract the first and second component of pairs.

```haskell
fst (1, 2) -- evaluates to 1
snd (1, 2) -- evaluates to 2
```

Or use pattern matching.
Pattern matching also works for tuples with more than two components.

case (1, 2, 3) of (result, _, _) => result -- evaluates to 1
case (1, 2, 3) of (_, result, _) => result -- evaluates to 2
case (1, 2, 3) of (_, _, result) => result -- evaluates to 3

Haskell does not provide standard functions like `fst` or `snd` for tuples with more than two components. The `tuple` library on Hackage provides such functions in the `Data.Tuple.Select` module.

Apply a binary function to a tuple (uncurrying)

Use the `uncurry` function (from Prelude or Data.Tuple) to convert a binary function to a function on tuples.

```
uncurry (+) (1, 2) -- computes 3
uncurry map (negate, [1, 2, 3]) -- computes [-1, -2, -3]
uncurry uncurry ((+), (1, 2)) -- computes 3
map (uncurry (+)) [(1, 2), (3, 4), (5, 6)] -- computes [3, 7, 11]
uncurry (curry f) -- computes the same as f
```

Apply a tuple function to two arguments (currying)

Use the `curry` function (from Prelude or Data.Tuple) to convert a function that takes tuples to a function that takes two arguments.

```
curry fst 1 2 -- computes 1
curry snd 1 2 -- computes 2
curry (uncurry f) -- computes the same as f
import Data.Tuple (swap)
curry swap 1 2 -- computes (2, 1)
```

Swap pair components

Use `swap` (from Data.Tuple) to swap the components of a pair.

```
import Data.Tuple (swap)
swap (1, 2) -- evaluates to (2, 1)
```
Or use pattern matching.

| case (1, 2) of (x, y) => (y, x) -- evaluates to (2, 1) |

**Strictness of matching a tuple**

The pattern \((p_1, p_2)\) is strict in the outermost tuple constructor, which can lead to unexpected strictness behaviour. For example, the following expression diverges (using \texttt{Data.Function.fix}):

\[
\text{fix} \; \$ \; \langle x, y \rangle \rightarrow (1, 2)
\]

since the match on \((x, y)\) is strict in the tuple constructor. However, the following expression, using an irrefutable pattern, evaluates to \((1, 2)\) as expected:

\[
\text{fix} \; \$ \; \sim \langle x, y \rangle \rightarrow (1, 2)
\]

Read Tuples (Pairs, Triples, ...) online: https://riptutorial.com/haskell/topic/5342/tuples--pairs--triples------
Chapter 69: Type algebra

Examples

Natural numbers in type algebra

We can draw a connection between the Haskell types and the natural numbers. This connection can be made assigning to every type the number of inhabitants it has.

Finite union types

For finite types, it suffices to see that we can assign a natural type to every number, based in the number of constructors. For example:

```haskell
type Color = Red | Yellow | Green
```

would be 3. And the `Bool` type would be 2.

```haskell
type Bool = True | False
```

Uniqueness up to isomorphism

We have seen that multiple types would correspond to a single number, but in this case, they would be isomorphic. This is to say that there would be a pair of morphisms \( f \) and \( g \), whose composition would be the identity, connecting the two types.

\[
f :: a \rightarrow b \\
g :: b \rightarrow a \\
f \cdot g = id = g \cdot f
\]

In this case, we would say that the types are isomorphic. We will consider two types equal in our algebra as long as they are isomorphic.

For example, two different representations of the number two are trivially isomorphic:

```haskell
type Bit  = I    | O 

type Bool = True | False

bitValue :: Bit -> Bool 
bitValue I = True 
bitValue O = False 

booleanBit :: Bool -> Bit 
booleanBit True  = I 
booleanBit False = O
```
Because we can see `bitValue . booleanBit == id == booleanBit . bitValue`

---

**One and Zero**

The representation of the number 1 is obviously a type with only one constructor. In Haskell, this type is canonically the type `()`, called Unit. Every other type with only one constructor is isomorphic to `()`.  

And our representation of 0 will be a type without constructors. This is the Void type in Haskell, as defined in `Data.Void`. This would be equivalent to a unhabited type, without data constructors:

```haskell
data Void
```

**Addition and multiplication**

The addition and multiplication have equivalents in this type algebra. They correspond to the tagged unions and product types.

```haskell
data Sum a b = A a | B b  
data Prod a b = Prod a b
```

We can see how the number of inhabitants of every type corresponds to the operations of the algebra.

Equivalently, we can use Either and (,) as type constructors for the addition and the multiplication. They are isomorphic to our previously defined types:

```haskell
type Sum' a b = Either a b  
type Prod' a b = (a,b)
```

The expected results of addition and multiplication are followed by the type algebra up to isomorphism. For example, we can see an isomorphism between 1 + 2, 2 + 1 and 3; as 1 + 2 = 3 = 2 + 1.

```haskell
data Color = Red | Green | Blue

f :: Sum () Bool -> Color
f (Left ())     = Red
f (Right True)  = Green
f (Right False) = Blue

g :: Color -> Sum () Bool
g Red   = Left ()
g Green = Right True
g Blue  = Right False

f' :: Sum Bool () -> Color
f' (Right ()) = Red
f' (Left True) = Green
f' (Left False) = Blue
```

https://riptutorial.com/
g' :: Color -> Sum Bool ()
g' Red   = Right ()
g' Green = Left True
g' Blue  = Left False

---

Rules of addition and multiplication

The common rules of commutativity, associativity and distributivity are valid because there are trivial isomorphisms between the following types:

```haskell
-- Commutativity
Sum a b           <=> Sum b a
Prod a b          <=> Prod b a
-- Associativity
Sum (Sum a b) c   <=> Sum a (Sum b c)
Prod (Prod a b) c <=> Prod a (Prod b c)
-- Distributivity
Prod a (Sum b c)  <=> Sum (Prod a b) (Prod a c)
```

Recursive types

---

Lists

Lists can be defined as:

```haskell
data List a = Nil | Cons a (List a)
```

If we translate this into our type algebra, we get

\[
\text{List}(a) = 1 + a \times \text{List}(a)
\]

But we can now substitute \( \text{List}(a) \) again in this expression multiple times, in order to get:

\[
\text{List}(a) = 1 + a + a^2 + a^3 + a^4 + ... \]

This makes sense if we see a list as a type that can contain only one value, as in \([\ ]\); or every value of type \(a\), as in \([x]\); or two values of type \(a\), as in \([x,y]\); and so on. The theoretical definition of List that we should get from there would be:

```haskell
-- Not working Haskell code!
data List a = Nil |
            | One a |
            | Two a a |
            | Three a a a |
            ... |
```

Trees

https://riptutorial.com/
We can do the same thing with binary trees, for example. If we define them as:

```haskell
data Tree a = Empty | Node a (Tree a) (Tree a)
```

We get the expression:

\[
\text{Tree}(a) = 1 + a \cdot \text{Tree}(a) \cdot \text{Tree}(a)
\]

And if we make the same substitutions again and again, we would obtain the following sequence:

\[
\text{Tree}(a) = 1 + a + 2 (a \cdot a) + 5 (a \cdot a \cdot a) + 14 (a \cdot a \cdot a \cdot a) + \ldots
\]

The coefficients we get here correspond to the Catalan numbers sequence, and the \(n\)-th catalan number is precisely the number of possible binary trees with \(n\) nodes.

**Derivatives**

The derivative of a type is the type of its type of one-hole contexts. This is the type that we would get if we make a type variable disappear in every possible point and sum the results.

As an example, we can take the triple type \((a,a,a)\), and derive it, obtaining

```haskell
data OneHoleContextsOfTriple = (a,a,()) | (a,(),a) | ((),a,a)
```

This is coherent with our usual definition of derivation, as:

\[
\frac{d}{da} (a \cdot a \cdot a) = 3 \cdot a \cdot a
\]

More on this topic can be read on this article.

**Functions**

Functions can be seen as exponentials in our algebra. As we can see, if we take a type \(a\) with \(n\) instances and a type \(b\) with \(m\) instances, the type \(a \to b\) will have \(m^k\) instances.

As an example, \(\text{Bool} \to \text{Bool}\) is isomorphic to \((\text{Bool},\text{Bool})\), as \(2^2 = 2^2\).

```haskell
iso1 :: (Bool -> Bool) -> (Bool,Bool)
iso1 f = (f True,f False)

iso2 :: (Bool,Bool) -> (Bool -> Bool)
iso2 (x,y) = (\p -> if p then x else y)
```

Read Type algebra online: https://riptutorial.com/haskell/topic/4905/type-algebra
Chapter 70: Type Application

Introduction

TypeApplications are an alternative to type annotations when the compiler struggles to infer types for a given expression.

This series of examples will explain the purpose of the TypeApplications extension and how to use it.

Don't forget to enable the extension by placing `{-# LANGUAGE TypeApplications #-}` at the top of your source file.

Examples

Avoiding type annotations

We use type annotations to avoid ambiguity. Type applications can be used for the same purpose. For example

```haskell
x :: Num a => a
x = 5

main :: IO ()
main = print x
```

This code has an ambiguity error. We know that `a` has a `Num` instance, and in order to print it we know it needs a `Show` instance. This could work if `a` was, for example, an `Int`, so to fix the error we can add a type annotation

```haskell
main = print (x :: Int)
```

Another solution using type applications would look like this

```haskell
main = print @Int x
```

To understand what this means we need to look at the type signature of `print`.

```haskell
print :: Show a => a -> IO ()
```

The function takes one parameter of type `a`, but another way to look at it is that it actually takes two parameters. The first one is a type parameter, the second one is a value whose type is the first parameter.

The main difference between value parameters and the type parameters is that the latter ones are implicitly provided to functions when we call them. Who provides them? The type inference
algorithm! What TypeApplications let us do is give those type parameters explicitly. This is especially useful when the type inference can't determine the correct type.

So to break down the above example

```haskell
print :: Show a => a -> IO ()
print @Int :: Int -> IO ()
print @Int x :: IO ()
```

**Type applications in other languages**

If you're familiar with languages like Java, C# or C++ and the concept of generics/templates then this comparison might be useful for you.

Say we have a generic function in C#

```csharp
public static T DoNothing<T>(T in) { return in; }
```

To call this function with a `float` we can do `DoNothing(5.0f)` or if we want to be explicit we can say `DoNothing<float>(5.0f)`. That part inside of the angle brackets is the type application.

In Haskell it's the same, except that the type parameters are not only implicit at call sites but also at definition sites.

```haskell
doNothing :: a -> a
doNothing x = x
```

This can also be made explicit using either **ScopedTypeVariables**, **Rank2Types** or **RankNTypes** extensions like this.

```haskell
doNothing :: forall a. a -> a
doNothing x = x
```

Then at the call site we can again either write `doNothing 5.0` or `doNothing @Float 5.0`

**Order of parameters**

The problem with type arguments being implicit becomes obvious once we have more than one. Which order do they come in?

```haskell
const :: a -> b -> a
```

Does writing `const @Int` mean `a` is equal to `Int`, or is it `b`? In case we explicitly state the type parameters using `forall` like `const :: forall a b. a -> b -> a` then the order is as written: `a`, then `b`.

If we don't, then the order of variables is from left to right. The first variable to be mentioned is the first type parameter, the second is the second type parameter and so on.
What if we want to specify the second type variable, but not the first? We can use a wildcard for the first variable like this

```haskell
const @_ @Int
```

The type of this expression is

```haskell
const @_ @Int :: a -> Int -> a
```

**Interaction with ambiguous types**

Say you're introducing a class of types that have a size in bytes.

```haskell
class SizeOf a where
    sizeOf :: a -> Int
```

The problem is that the size should be constant for every value of that type. We don't actually want the `sizeOf` function to depend on `a`, but only on its type.

Without type applications, the best solution we had was the `Proxy` type defined like this

```haskell
data Proxy a = Proxy
```

The purpose of this type is to carry type information, but no value information. Then our class could look like this

```haskell
class SizeOf a where
    sizeOf :: Proxy a -> Int
```

Now you might be wondering, why not drop the first argument altogether? The type of our function would then just be `sizeOf :: Int` or, to be more precise because it is a method of a class, `sizeOf :: SizeOf a -> Int` or to be even more explicit `sizeOf :: forall a. SizeOf a -> Int`.

The problem is type inference. If I write `sizeOf` somewhere, the inference algorithm only knows that I expect an `Int`. It has no idea what type I want to substitute for `a`. Because of this, the definition gets rejected by the compiler *unless* you have the `{-# LANGUAGE AllowAmbiguousTypes #-}` extension enabled. In that case the definition compiles, it just can't be used anywhere without an ambiguity error.

Luckily, the introduction of type applications saves the day! Now we can write `sizeOf @Int`, explicitly saying that `a` is `Int`. Type applications allow us to provide a type parameter, even if it doesn't appear in the *actual parameters of the function!*

Read Type Application online: [https://riptutorial.com/haskell/topic/10767/type-application](https://riptutorial.com/haskell/topic/10767/type-application)
Chapter 71: Type Classes

Introduction

Typeclasses in Haskell are a means of defining the behaviour associated with a type separately from that type's definition. Whereas, say, in Java, you'd define the behaviour as part of the type's definition -- i.e. in an interface, abstract class or concrete class -- Haskell keeps these two things separate.

There are a number of typeclasses already defined in Haskell's base package. The relationship between these is illustrated in the Remarks section below.

Remarks

The following diagram taken from the Typeclassopedia article shows the relationship between the various typeclasses in Haskell.

Examples

Maybe and the Functor Class

In Haskell, data types can have arguments just like functions. Take the Maybe type for example.

Maybe is a very useful type which allows us to represent the idea of failure, or the possibility thereof. In other words, if there is a possibility that a computation will fail, we use the Maybe type there. Maybe acts kind of like a wrapper for other types, giving them additional functionality.

Its actual declaration is fairly simple.

```haskell
Maybe a = Just a | Nothing
```
What this tells is that a `Maybe` comes in two forms, a `Just`, which represents success, and a `Nothing`, which represents failure. `Just` takes one argument which determines the type of the `Maybe`, and `Nothing` takes none. For example, the value `Just "foo"` will have type `Maybe String`, which is a string type wrapped with the additional `Maybe` functionality. The value `Nothing` has type `Maybe a` where `a` can be any type.

This idea of wrapping types to give them additional functionality is a very useful one, and is applicable to more than just `Maybe`. Other examples include the `Either`, `IO` and list types, each providing different functionality. However, there are some actions and abilities which are common to all of these wrapper types. The most notable of these is the ability to modify the encapsulated value.

It is common to think of these kinds of types as boxes which can have values placed in them. Different boxes hold different values and do different things, but none are useful without being able to access the contents within.

To encapsulate this idea, Haskell comes with a standard typeclass, named `Functor`. It is defined as follows.

```haskell
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

As can be seen, the class has a single function, `fmap`, of two arguments. The first argument is a function from one type, `a`, to another, `b`. The second argument is a functor (wrapper type) containing a value of type `a`. It returns a functor (wrapper type) containing a value of type `b`.

In simple terms, `fmap` takes a function and applies to the value inside of a functor. It is the only function necessary for a type to be a member of the `Functor` class, but it is extremely useful. Functions operating on functors that have more specific applications can be found in the `Applicative` and `Monad` typeclasses.

**Type class inheritance: Ord type class**

Haskell supports a notion of class extension. For example, the class `Ord` inherits all of the operations in `Eq`, but in addition has a `compare` function that returns an `Ordering` between values. `Ord` may also contain the common order comparison operators, as well as a `min` method and a `max` method.

The `->` notation has the same meaning as it does in a function signature and requires type `a` to implement `Eq`, in order to implement `Ord`.

```haskell
data Ordering = EQ | LT | GT

class Eq a => Ord a where
  compare :: Ord a -> a -> a -> Ordering
  (<)   :: Ord a -> a -> a -> Bool
  (<=)  :: Ord a -> a -> a -> Bool
  (>)   :: Ord a -> a -> a -> Bool
  (>=)  :: Ord a -> a -> a -> Bool
  min   :: Ord a -> a -> a -> a
```
All of the methods following `compare` can be derived from it in a number of ways:

```
x < y   = compare x y == LT
x <= y  = x < y || x == y -- Note the use of (==) inherited from Eq
x > y   = not (x <= y)
x >= y  = not (x < y)
```

```
min x y = case compare x y of
         EQ -> x
         LT -> x
         GT -> y
```

```
max x y = case compare x y of
         EQ -> x
         LT -> y
         GT -> x
```

Type classes that themselves extend `Ord` must implement at least either the `compare` method or the `<=` method themselves, which builds up the directed inheritance lattice.

**Eq**

All basic datatypes (like `Int`, `String`, `Eq a => [a]`) from Prelude except for functions and `IO` have instances of `Eq`. If a type instantiates `Eq` it means that we know how to compare two values for *value* or *structural* equality.

```
> 3 == 2
False
> 3 == 3
True
```

### Required methods

- `(==) :: Eq a -> a -> a -> Boolean` or `(/=) :: Eq a -> a -> a -> Boolean` (if only one is implemented, the other defaults to the negation of the defined one)

### Defines

- `(==) :: Eq a -> a -> a -> Boolean`
- `(/=) :: Eq a -> a -> a -> Boolean`

### Direct superclasses

None
Notable subclasses

- Ord

Ord

Types instantiating Ord include, e.g., Int, String, and [a] (for types a where there's an Ord a instance). If a type instantiates Ord it means that we know a "natural" ordering of values of that type. Note, there are often many possible choices of the “natural” ordering of a type and Ord forces us to favor one.

Ord provides the standard \((\leq), (\lt), (\gt), (\geq)\) operators but interestingly defines them all using a custom algebraic data type

```haskell
data Ordering = LT | EQ | GT

compare :: Ord a => a -> a -> Ordering
```

Required methods

- `compare :: Ord a => a -> a -> Ordering` or `(\leq) :: Ord a => a -> a -> Boolean` (the standard's default `compare` method uses `(\leq)` in its implementation)

Defines

- `compare :: Ord a => a -> a -> Ordering`
- `(\leq) :: Ord a => a -> a -> Boolean`
- `(\lt) :: Ord a => a -> a -> Boolean`
- `(\gt) :: Ord a => a -> a -> Boolean`
- `(\geq) :: Ord a => a -> a -> Boolean`
- `min :: Ord a => a -> a -> a`
- `max :: Ord a => a -> a -> a`

Direct superclasses

- Eq

Monoid

Types instantiating Monoid include lists, numbers, and functions with Monoid return values, among others. To instantiate Monoid a type must support an associative binary operation (mappend or `(<>`) which combines its values, and have a special "zero" value (mempty) such that combining a value with it does not change that value:
Intuitively, `Monoid` types are "list-like" in that they support appending values together. Alternatively, `Monoid` types can be thought of as sequences of values for which we care about the order but not the grouping. For instance, a binary tree is a `Monoid`, but using the `Monoid` operations we cannot witness its branching structure, only a traversal of its values (see `Foldable` and `Traversable`).

### Required methods

- `mempty :: Monoid m => m`
- `mappend :: Monoid m => m -> m -> m`

### Direct superclasses

None

**Num**

The most general class for number types, more precisely for rings, i.e. numbers that can be added and subtracted and multiplied in the usual sense, but not necessarily divided.

This class contains both integral types (Int, Integer, Word32 etc.) and fractional types (Double, Rational, also complex numbers etc.). In case of finite types, the semantics are generally understood as modular arithmetic, i.e. with over- and underflow.

Note that the rules for the numerical classes are much less strictly obeyed than the monad or monoid laws, or those for equality comparison. In particular, floating-point numbers generally obey laws only in a approximate sense.

### The methods

- `fromInteger :: Num a => Integer -> a`. convert an integer to the general number type (wrapping around the range, if necessary). Haskell number literals can be understood as a monomorphic `Integer` literal with the general conversion around it, so you can use the literal 5 in both an `Int` context and a `Complex Double` setting.

- `(+) :: Num a => a -> a -> a`. Standard addition, generally understood as associative and commutative, i.e.,

  ```plaintext
  a + (b + c) = (a + b) + c  
  a + b = b + a
  ```

- `(-) :: Num a => a -> a -> a`. Subtraction, which is the inverse of addition:
\[(a - b) + b = (a + b) - b = a\]

- \((*) :: \text{Num } a \rightarrow a \rightarrow a\). Multiplication, an associative operation that’s distributive over addition:

\[
a * (b * c) = (a * b) * c
a * (b + c) = a * b + a * c
\]

for the most common instances, multiplication is also commutative, but this is definitely not a requirement.

- \(\text{negate} :: \text{Num } a \rightarrow a \rightarrow a\). The full name of the unary negation operator. \(-1\) is syntactic sugar for \(\text{negate 1}\).

\[-a = \text{negate } a = 0 - a\]

- \(\text{abs} :: \text{Num } a \rightarrow a \rightarrow a\). The absolute-value function always gives a non-negative result of the same magnitude

\[
\text{abs } (-a) = \text{abs } a
\text{abs } (\text{abs } a) = \text{abs } a
\]

\(\text{abs } a = 0\) should only happen if \(a = 0\).

For \text{real} types it’s clear what non-negative means: you always have \(\text{abs } a >= 0\). Complex etc. types don’t have a well-defined ordering, however the result of \(\text{abs}\) should always lie in the real subset\(^\dagger\) (i.e. give a number that could also be written as a single number literal without negation).

- \(\text{signum} :: \text{Num } a \rightarrow a \rightarrow a\). The sign function, according to the name, yields only \(-1\) or \(1\), depending on the sign of the argument. Actually, that’s only true for nonzero real numbers; in general \(\text{signum}\) is better understood as the \textit{normalising} function:

\[
\text{abs } (\text{signum } a) = 1 \quad \text{-- unless } a=0
\text{signum } a * \text{abs } a = a \quad \text{-- This is required to be true for all Num instances}
\]

Note that section 6.4.4 of the Haskell 2010 Report explicitly requires this last equality to hold for any valid \text{Num} instance.

Some libraries, notably \texttt{linear} and \texttt{hmatrix}, have a much laxer understanding of what the \texttt{Num} class is for: they treat it just as a \textit{way to overload the arithmetic operators}. While this is pretty straightforward for \(+\) and \(-\), it already becomes troublesome with \(-\) and more so with the other methods. For instance, \textit{should} \(-\) \textit{mean matrix multiplication or element-wise multiplication}?
It is arguably a bad idea to define such non-number instances; please consider dedicated classes such as \texttt{VectorSpace}.
† In particular, the “negatives” of unsigned types are wrapped around to large positive, e.g. 
\((-4 :: \text{Word32}) == 4294967292.\)

‡ This is widely not fulfilled: vector types do not have a real subset. The controversial \texttt{Num}-instances for such types generally define \texttt{abs} and \texttt{signum} element-wise, which mathematically speaking doesn’t really make sense.

Read Type Classes online: https://riptutorial.com/haskell/topic/1879/type-classes
Chapter 72: Type Families

Examples

Type Synonym Families

Type synonym families are just type-level functions: they associate parameter types with result types. These come in three different varieties.

Closed type-synonym families

These work much like ordinary value-level Haskell functions: you specify some clauses, mapping certain types to others:

```
{-# LANGUAGE TypeFamilies #-}
type family Vanquisher a where
  Vanquisher Rock = Paper
  Vanquisher Paper = Scissors
  Vanquisher Scissors = Rock

data Rock=Rock; data Paper=Paper; data Scissors=Scissors
```

Open type-synonym families

These work more like typeclass instances: anybody can add more clauses in other modules.

```
type family DoubledSize w

  type instance DoubledSize Word16 = Word32
  type instance DoubledSize Word32 = Word64

  -- Other instances might appear in other modules, but two instances cannot overlap
  -- in a way that would produce different results.
```

Class-associated type synonyms

An open type family can also be combined with an actual class. This is usually done when, like with associated data families, some class method needs additional helper objects, and these helper objects can be different for different instances but may possibly also shared. A good example is VectorSpace class:

```
class VectorSpace v where
  type Scalar v :: *
  (*^) :: Scalar v -> v -> v
```
instance VectorSpace Double where  
  type Scalar Double = Double  
  μ *^ n = μ * n  

instance VectorSpace (Double,Double) where  
  type Scalar (Double,Double) = Double  
  μ *^ (n,m) = (μ*n, μ*m)  

instance VectorSpace (Complex Double) where  
  type Scalar (Complex Double) = Complex Double  
  μ *^ n = μ*n  

Note how in the first two instances, the implementation of `Scalar` is the same. This would not be possible with an associated data family: data families are **injective**, type-synonym families aren't.

While non-injectivity opens up some possibilities like the above, it also makes type inference more difficult. For instance, the following will not typecheck:

```haskell
class Foo a where  
  type Bar a :: *  
  bar :: a -> Bar a  
instance Foo Int where  
  type Bar Int = String  
  bar = show  
instance Foo Double where  
  type Bar Double = Bool  
  bar = (>0)  
main = putStrLn (bar 1)
```

In this case, the compiler can't know what instance to use, because the argument to `bar` is itself just a polymorphic `Num` literal. And the type function `Bar` can't be resolved in “inverse direction”, precisely because it's not injective† and hence not invertible (there could be more than one type with `Bar a = String`).

†With only these two instances, it *is* actually injective, but the compiler can't know somebody won't add more instances later on and thereby break the behaviour.

**Datatype Families**

Data families can be used to build datatypes that have different implementations based on their type arguments.

**Standalone data families**

```haskell
{-# LANGUAGE TypeFamilies #-}  
data family List a  
data instance List Char = Nil | Cons Char (List Char)  
data instance List () = UnitList Int
```

In the above declaration, `Nil :: List Char`, and `UnitList :: Int -> List ()`
Associated data families

Data families can also be associated with typeclasses. This is often useful for types with “helper objects”, which are required for generic typeclass methods but need to contain different information depending on the concrete instance. For instance, indexing locations in a list just requires a single number, whereas in a tree you need a number to indicate the path at each node:

```haskell
class Container f where
data Location f
get :: Location f -> f a -> Maybe a

instance Container [] where
data Location [] = ListLoc Int
get (ListLoc i) xs
  | i < length xs = Just $ xs!!i
  | otherwise = Nothing

instance Container Tree where
data Location Tree = ThisNode | NodePath Int (Location Tree)
get ThisNode (Node x _) = Just x
get (NodePath i path) (Node _ sfo) = get path =<< get i sfo
```

Injectivity

Type Families are not necessarily injective. Therefore, we cannot infer the parameter from an application. For example, in servant, given a type Server a we cannot infer the type a. To solve this problem, we can use Proxy. For example, in servant, the serve function has type ... Proxy a -> Server a -> .... We can infer a from Proxy a because Proxy is defined by data which is injective.

Read Type Families online: https://riptutorial.com/haskell/topic/2955/type-families
Chapter 73: Typed holes

Remarks

One of the strengths of Haskell is the ability to leverage the type system to model parts of your problem domain in the type system. In doing so, one often encounters very complex types. When writing programs with these types (i.e. with values having these types) it occasionally becomes nearly unmangeable to 'juggle' all of the types. As of GHC 7.8, there is a new syntactic feature called typed holes. Typed holes do not change the semantics of the core language; they are intended purely as an aid for writing programs.

For an in-depth explanation of typed holes, as well as a discussion of the design of typed holes, see the Haskell wiki.

Section of the GHC user guide on typed holes.

Examples

Syntax of typed holes

A typed hole is a single underscore (_) or a valid Haskell identifier which is not in scope, in an expression context. Before the existance of typed holes, both of these things would trigger an error, so the new syntax does not interfere with any old syntax.

Controlling behaviour of typed holes

The default behaviour of typed holes is to produce a compile-time error when encountering a typed hole. However, there are several flags to fine-tune their behaviour. These flags are summarized as follows (GHC trac):

- **By default GHC has typed holes enabled and produces a compile error when it encounters a typed hole.**

- **When `-fdefer-type-errors` or `-fdefer-typed-holes` is enabled, hole errors are converted to warnings and result in runtime errors when evaluated.**

- **The warning flag `-fwarn-typed-holes` is on by default. Without `-fdefer-type-errors` or `-fdefer-typed-holes` this flag is a no-op, since typed holes are an error under these conditions. If either of the defer flags are enabled (converting typed hole errors into warnings) the `-fno-warn-typed-holes` flag disables the warnings. This means compilation silently succeeds and evaluating a hole will produce a runtime error.**

Semantics of typed holes
The value of a type hole can simply said to be undefined, although a typed hole triggers a compile-time error, so it is not strictly necessary to assign it a value. However, a typed hole (when they are enabled) produces a compile time error (or warning with deferred type errors) which states the name of the typed hole, its inferred most general type, and the types of any local bindings. For example:

```
Prelude> \x -> _var + length (drop 1 x)

<interactive>:19:7: Warning:
  Found hole '_var' with type: Int
  Relevant bindings include
  x :: [a] (bound at <interactive>:19:2)
  it :: [a] -> Int (bound at <interactive>:19:1)
  In the first argument of '(+)', namely '_var'
  In the expression: _var + length (drop 1 x)
  In the expression: \ x -> _var + length (drop 1 x)
```

Note that in the case of typed holes in expressions entered into the GHCi repl (as above), the type of the expression entered also reported, as it (here of type `[a] -> Int`).

Using typed holes to define a class instance

Typed holes can make it easier to define functions, through an interactive process.

Say you want to define a class instance `Foo Bar` (for your custom `Bar` type, in order to use it with some polymorphic library function that requires a `Foo` instance). You would now traditionally look up the documentation of `Foo`, figure out which methods you need to define, scrutinise their types etc. – but with typed holes, you can actually skip that!

First just define a dummy instance:

```
instance Foo Bar where
```

The compiler will now complain

```
Bar.hs:13:10: Warning:
  No explicit implementation for
  'foom' and 'quun'
  In the instance declaration for 'Foo Bar'
```

Ok, so we need to define `foom` for `Bar`. But what is that even supposed to be? Again we’re too lazy to look in the documentation, and just ask the compiler:

```
instance Foo Bar where
  foom = _
```

Here we’ve used a typed hole as a simple “documentation query”. The compiler outputs

```
Bar.hs:14:10:
  Found hole '_' with type: Bar -> Gronk Bar
```
Note how the compiler has already filled the class type variable with the concrete type `Bar` that we want to instantiate it for. This can make the signature a lot easier to understand than the polymorphic one found in the class documentation, especially if you’re dealing with a more complicated method of e.g. a multi-parameter type class.

But what the hell is `Gronk`? At this point, it is probably a good idea to ask Hayoo. However we may still get away without that: as a blind guess, we assume that this is not only a type constructor but also the single value constructor, i.e. it can be used as a function that will somehow produce a `Gronk a` value. So we try

```haskell
instance Foo Bar where
    foom bar = _ Gronk
```

If we’re lucky, `Gronk` is actually a value, and the compiler will now say

```haskell
Found hole ‘_’
    with type: (Int -> [(Int, b0)]) -> Gronk b0 -> Gronk Bar
Where: ‘b0’ is an ambiguous type variable
```

Ok, that’s ugly – at first just note that `Gronk` has two arguments, so we can refine our attempt:

```haskell
instance Foo Bar where
    foom bar = Gronk _ _
```

And this now is pretty clear:

```haskell
Found hole ‘_’ with type: [(Int, Bar)]
Relevant bindings include
    bar :: Bar (bound at Bar.hs:14:29)
    foom :: Bar -> Gronk Bar (bound at Foo.hs:15:24)
In the second argument of ‘Gronk’, namely ‘_’
In the expression: Gronk _ _
In an equation for ‘foom’: foom bar = Gronk _ _
```

You can now further progress by e.g. deconstructing the `bar` value (the components will then show up, with types, in the relevant bindings section). Often, it is at some point completely obvious what the correct definition will be, because you you see all available arguments and the types fit together like a jigsaw puzzle. Or alternatively, you may see that the definition is impossible and why.

All of this works best in an editor with interactive compilation, e.g. Emacs with haskell-mode. You can then use typed holes much like mouse-over value queries in an IDE for an interpreted dynamic imperative language, but without all the limitations.
Read Typed holes online: https://riptutorial.com/haskell/topic/4913/typed-holes
Chapter 74: Using GHCi

Remarks

GHCI is the interactive REPL that comes bundled with GHC.

Examples

Starting GHCi

Type `ghci` at a shell prompt to start GHCI.

```
$ ghci
GHCi, version 8.0.1: http://www.haskell.org/ghc/  :? for help
Prelude>
```

Changing the GHCI default prompt

By default, GHCI’s prompt shows all the modules you have loaded into your interactive session. If you have many modules loaded this can get long:

```
Prelude Data.List Control.Monad> -- etc
```

The `:set prompt` command changes the prompt for this interactive session.

```
Prelude Data.List Control.Monad> :set prompt "foo> "
foo>
```

To change the prompt permanently, add `:set prompt "foo> "` to the GHCI config file.

The GHCI configuration file

GHCI uses a configuration file in `~/.ghci`. A configuration file consists of a sequence of commands which GHCI will execute on startup.

```
$ echo ":set prompt \"foo> \"" > ~/.ghci
$ ghci
GHCi, version 8.0.1: http://www.haskell.org/ghc/  :? for help
Loaded GHCI configuration from ~/.ghci
foo>
```

Loading a file

The `:l` or `:load` command type-checks and loads a file.
Quitting GHCi

You can quit GHCi simply with :q or :quit

ghci> :q
Leaving GHCi.

ghci> :quit
Leaving GHCi.

Alternatively, the shortcut CTRL+D (Cmd+d for OSX) has the same effect as :q.

Reloading a already loaded file

If you have loaded a file into GHCi (e.g. using :l filename.hs) and you have changed the file in an editor outside of GHCi you must reload the file with :r or :reload in order to make use of the changes, hence you don’t need to type again the filename.

ghci> :r
OK, modules loaded: Main.

ghci> :reload
OK, modules loaded: Main.

Breakpoints with GHCi

GHCi supports imperative-style breakpoints out of the box with interpreted code (code that's been :loaded).

With the following program:

```hs
-- mySum.hs
doSum n = do
  putStrLn ("Counting to " ++ (show n))
  let v = sum [1..n]
  putStrLn ("sum to " ++ (show n) ++ " = " ++ (show v))
```

loaded into GHCi:

Prelude> :load mySum.hs
[1 of 1] Compiling Main       ( mySum.hs, interpreted )
Ok, modules loaded: Main.
*Main>
We can now set breakpoints using line numbers:

```haskell
*Main> :break 2
Breakpoint 0 activated at mySum.hs:2:3-39
```

and GHCi will stop at the relevant line when we run the function:

```haskell
*Main> doSum 12
Stopped at mySum.hs:2:3-39
_result :: IO () = _
n :: Integer = 12
[mySum.hs:2:3-39] *Main>
```

It might be confusing where we are in the program, so we can use :list to clarify:

```haskell
[mySum.hs:2:3-39] *Main> :list
1  doSum n = do
2    putStrLn ("Counting to " ++ (show n))  -- GHCi will emphasise this line, as that's where we've stopped
3    let v = sum [1..n]

We can print variables, and continue execution too:

```haskell
[mySum.hs:2:3-39] *Main> n
12
:continue
Counting to 12
sum to 12 = 78
*Main>
```

Multi-line statements

The : { instruction begins multi-line mode and : } ends it. In multi-line mode GHCi will interpret newlines as semicolons, not as the end of an instruction.

```haskell
ghci> :{
ghci| myFoldr f z [] = z
ghci| myFoldr f z (y:ys) = f y (myFoldr f z ys)
ghci|
ghci> :t myFoldr
myFoldr :: (a -> b -> b) -> b -> [a] -> b
```

Read Using GHCi online: https://riptutorial.com/haskell/topic/3407/using-ghci
Chapter 75: Vectors

Remarks

It [Data.Vector] has an emphasis on very high performance through loop fusion, whilst retaining a rich interface. The main data types are boxed and unboxed arrays, and arrays may be immutable (pure), or mutable. Arrays may hold Storable elements, suitable for passing to and from C, and you can convert between the array types. Arrays are indexed by non-negative Int values.

The Haskell Wiki has these recommendations:

In general:

- End users should use Data.Vector.Unboxed for most cases
- If you need to store more complex structures, use Data.Vector
- If you need to pass to C, use Data.Vector.Storable

For library writers:

- Use the generic interface, to ensure your library is maximally flexible:
  Data.Vector.Generic

Examples

The Data.Vector Module

The Data.Vector module provided by the vector is a high performance library for working with arrays.

Once you've imported Data.Vector, it's easy to start using a Vector:

```
Prelude> import Data.Vector
Prelude Data.Vector> let a = fromList [2,3,4]
```

```
Prelude Data.Vector> :t a
a :: Vector Integer
```

You can even have a multi-dimensional array:

```
Prelude Data.Vector> let x = fromList [ fromList [1 .. x] | x <- [1..10] ]
Prelude Data.Vector> :t x
x :: Vector (Vector Integer)
```
Filtering a Vector

Filter odd elements:

```
Prelude Data.Vector> Data.Vector.filter odd y
fromList [1,3,5,7,9,11] :: Data.Vector.Vector
```

Mapping ("map") and Reducing ("fold") a Vector

Vectors can be `map`d and `fold`d, `filter`d and `zip`d:

```
Prelude Data.Vector> Data.Vector.map (^2) y
fromList [0,1,4,9,16,25,36,49,64,81,100,121] :: Data.Vector.Vector
```

Reduce to a single value:

```
Prelude Data.Vector> Data.Vector.foldl (+) 0 y
66
```

Working on Multiple Vectors

Zip two arrays into an array of pairs:

```
Prelude Data.Vector> Data.Vector.zip y y
fromList [(0,0),(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8),(9,9),(10,10),(11,11)] :: Data.Vector.Vector
```

Read Vectors online: https://riptutorial.com/haskell/topic/4738/vectors
Chapter 76: Web Development

Examples

Servant

Servant is a library for declaring APIs at the type-level and then:

- write servers (this part of servant can be considered a web framework),
- obtain client functions (in haskell),
- generate client functions for other programming languages,
- generate documentation for your web applications
- and more...

Servant has a concise yet powerful API. A simple API can be written in very few lines of code:

```haskell
{-# LANGUAGE DataKinds #-}
{-# LANGUAGE TypeOperators #-}

import Data.Text
import Data.Aeson.Types
import GHC.Generics
import Servant.API

data SortBy = Age | Name

instance ToJSON User  -- automatically convert User to JSON

Now we can declare our API:

```haskell

```

```haskell

Now we can define our handler:

```haskell

```

```haskell

which states that we wish to expose /users to GET requests with a query param sortby of type SortBy and return JSON of type User in the response.

Now we can define our handler:

```haskell

```

```haskell

```
And the main method which listens on port 8081 and serves our user API:

```haskell
main :: IO ()
main = run 8081 app1
```

Note, Stack has a template for generating basic APIs in Servant, which is useful for getting up and running very quick.

**Yesod**

Yesod project can be created with `stack new` using following templates:

- `yesod-minimal`. Simplest Yesod scaffold possible.
- `yesod-mongo`. Uses MongoDB as DB engine.
- `yesod-mysql`. Uses MySQL as DB engine.
- `yesod-postgres`. Uses PostgreSQL as DB engine.
- `yesod-simple`. Recommended template to use, if you don't need database.
- `yesod-sqlite`. Uses SQLite as DB engine.

`yesod-bin` package provides `yesod` executable, which can be used to run development server. Note that you also can run your application directly, so `yesod` tool is optional.

`Application.hs` contains code that dispatches requests between handlers. It also sets up database and logging settings, if you used them.

`Foundation.hs` defines `App` type, that can be seen as an environment for all handlers. Being in `HandlerT` monad, you can get this value using `getYesod` function.

`Import.hs` is a module that just re-exports commonly used stuff.

`Model.hs` contains Template Haskell that generates code and data types used for DB interaction. Present only if you are using DB.

`config/models` is where you define your DB schema. Used by `Model.hs`.

`config/routes` defines URI's of the Web application. For each HTTP method of the route, you'd need to create a handler named `{method}{RouteR}`.

`static/` directory contains site’s static resources. These get compiled into binary by `Settings/StaticFiles.hs` module.

`templates/` directory contains Shakespeare templates that are used when serving requests.

Finally, `Handler/` directory contains modules that define handlers for routes.

Each handler is a `HandlerT` monad action based on IO. You can inspect request parameters, its body and other information, make queries to the DB with `runDB`, perform arbitrary IO and return various types of content to the user. To serve HTML, `defaultLayout` function is used that allows neat composition of shakespearean templates.
Read Web Development online: https://riptutorial.com/haskell/topic/4721/web-development
Chapter 77: XML

Introduction

Encoding and decoding of XML documents.

Examples

Encoding a record using the `xml` library

```haskell
{-# LANGUAGE RecordWildCards #-}
import Text.XML.Light

data Package = Package
  { pOrderNo :: String
  , pOrderPos :: String
  , pBarcode :: String
  , pNumber :: String
  }

-- | Create XML from a Package
instance Node Package where
  node qn Package {..} =
    node qn
      [ unode "package_number" pNumber
      , unode "package_barcode" pBarcode
      , unode "order_number" pOrderNo
      , unode "order_position" pOrderPos
      ]
```

Read XML online: https://riptutorial.com/haskell/topic/9264/xml
Chapter 78: zipWithM

Introduction

zipWithM is to zipWith as mapM is to map: it lets you combine two lists using a monadic function.

From the module Control.Monad

Syntax

- zipWithM :: Applicative m => (a -> b -> m c) -> [a] -> [b] -> m [c]

Examples

Calculating sales prices

Suppose you want to see if a certain set of sales prices makes sense for a store.

The items originally cost $5, so you don't want to accept the sale if the sales price is less for any of them, but you do want to know what the new price is otherwise.

Calculating one price is easy: you calculate the sales price, and return Nothing if you don't get a profit:

```haskell
calculateOne :: Double -> Double -> Maybe Double
calculateOne price percent = let newPrice = price*(percent/100)
                           in if newPrice < 5 then Nothing else Just newPrice
```

To calculate it for the entire sale, zipWithM makes it really simple:

```haskell
calculateAllPrices :: [Double] -> [Double] -> Maybe [Double]
calculateAllPrices prices percents = zipWithM calculateOne prices percents
```

This will return Nothing if any of the sales prices are below $5.

Read zipWithM online: https://riptutorial.com/haskell/topic/9685/zipwithm
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